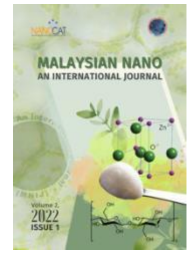
Malaysian NANO-An International Journal




**Research Article**

# Marangoni Convection in a Wavy Trapezoidal Enclosure with Nanofluids in Different Amplitudes

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Received June 10, 2022
Revised July 24, 2022
Accepted July 28, 2022

DOI:
https://doi.org/10.22452/mnij.vol2no1.4

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**Abstract**
In this paper, Marangoni convection inside a trapezoidal enclosure with nanofluid is numerically studied for different amplitudes. One side of the cavity is heated while the other is cooled with natural convection taking place at the top wall. The fluid flow and heat transfer are investigated at different amplitudes, particle volume fractions, Rayleigh and Marangoni numbers. The results are then shown in the form of streamlines, isotherms, local Nusselt and average Nusselt numbers. It was discovered that increasing the amplitude of the heated wavy wall improved heat transfer significantly, though the strength of fluid flow diminishes slightly.

**Keywords***:* Marangoni convection; Wavy Enclosure; Nanofluid; Heat Transfer

## 1. Introduction

Marangoni convection plays a significant role in many fields including biomedicine, engineering, and physics. It is also used extensively in artwork to dye flat surfaces. This phenomenon occurs because of the surface tension gradient being induced on the interface due to the change in temperature gradient. Many studies and investigations have been carried out involving the Marangoni convection with most being numerical where the natural convection of different types of fluid is examined in a cavity or enclosure under certain boundary conditions. Huang *et al.* conducted a study on flow and heat transfer of unsteady natural convection in a square enclosure with a cylindrical heat source in the centre for various values of $Ra$. It was found that the heat transfer rate increased with not only the Rayleigh number but also the amplitude of the heat source. Alhashash and Saleh also investigated natural convection of nanofluid in a square enclosure, but with wavy heated and cooled walls on the sides. It was discovered that increasing the amplitude of the wavy walls intensified both fluid flow and heat transfer. Roslan *et al*. performed a numerical study on natural convection in a square cavity with a heated cylindrical source. They concluded that the heat transfer performance increased when the source temperature signal's oscillation was augmented, which is in accordance with the investigation conducted by Huang *et al*. Hayat *et al.* mathematically analysed the effects of carbon nanotubes on Marangoni convection of nanofluid. It was found that both the temperature of the fluid and thermal boundary layer thickness increased as the volume fraction concentration was enhanced. Ellahi carried out a study on the effects of nanoparticle shape on Marangoni convection layer flow. They reported that particles which were spherical achieved the highest heat transfer rate. Das and Mahmud investigated fluid flow and heat transfer rate of fluid inside a rectangular wavy cavity. They discovered that increasing the amplitude caused the periodic behaviour of the Nusselt number distribution to change for when the Grashof number is big. However, when the Grashof number is small, the heat transfer rate improved as the amplitude wavelength-ratio increased. Hamzah *et al.*, Boulahla *et al.* and Sadeghi *et al*. conducted numerical studies of natural convection of nanofluid in a wavy enclosure. Hamzah *et al.* used a cavity with wavy walls on both ends of the enclosure and it was discovered that the Darcy number and amplitude influenced the heat transfer rate. Bouhlala *et al.*, on the other hand, had wavy walls on the top and bottom parts of the cavity rather than the sides. They found that if the amplitude was increased, the temperature of the fluid would drop. Sadeghi *et al.* carried out an investigation involving a slightly different cavity, though the top and bottom walls were wavy. They discovered that while the amplitude did influence the heat transfer rate, it wasn't as significant compared to other parameters such as the Rayleigh number. Roy *et al.* studied natural convection in a square geometry with a wavy wall obstacle set in the middle. He concluded that fluid flow became weaker as the amplitude and undulation were increased, though when it came to heat transfer, the Nusselt number augmented significantly. This is in accordance with a study conducted by Shirvan *et al.* where natural convection heat transfer was investigated in a wavy square enclosure. The findings showed that when the amplitude and wavelength were increased, the Nusselt number rose steadily. The strength of the vortex, however, was reduced. Job and Gunakala studied unsteady magnetohydrodynamic natural convection in a trapezoidal cavity with a wavy heated base. It was concluded that as the amplitude of the wavy heated wall increased, the flow circulation strength was reduced, and that the temperature of the wavy wall increased.

In this paper, Marangoni convection of nanofluid inside a wavy trapezoidal cavity with different amplitudes is numerically studied for various values of $Ra,\ Ma$ and $\phi$. While there have been studies on enclosures with wavy walls, there has been none involving trapezoidal cavities and hence, this investigation aims to bridge that gap by examining how heat transfer and fluid flow are affected when the amplitude of the heated wavy wall is differed.

## 2. Mathematical Formulation

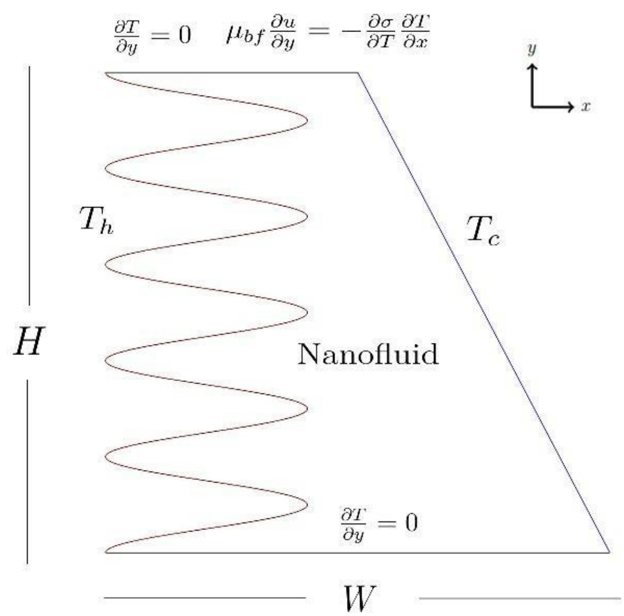



**Figure 1:** Two-dimensional physical model of the wavy trapezoidal enclosure

Figure 1 shows a physical model of the two-dimensional trapezoidal enclosure with a wavy side wall. The left wavy wall is described by

$$\boldsymbol{x = A \cdot (1 - \cos(\frac{2\pi\lambda y}{H}))} \tag{1}$$

The cavity has height H and width W, and both are assumed to be the same in this paper. The left side of the enclosure is heated at a fixed temperature while the right side is cooled under the same assumption, though clearly at a much lower temperature. The rest of the walls are adiabatic and fluid in the cavity is water-based nanofluid. The physical properties of the fluid and $Al_2O_3$nanoparticles are presented in Table 1. All values are assumed to be constant. The amplitude and wavelength of the wavy wall are denoted by $A$ and λ with the amplitude set to 0.1 and 0.2 for wavelength 1, 2 and 5.

This study assumes that the flow is laminar, incompressible, and steady, as well as taking into account that viscous dissipation is absent. Gravity acts in the vertically downward direction and the no-slip boundary condition is applied to the left, right and bottom walls. The slip boundary condition is set for the top wall where Marangoni convection takes place. The two-dimensional, dimensionless governing equations, boundary conditions and stream function are given by

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr\frac{\mu_{nf}/\mu_f}{\rho_{nf}/\rho_f}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{3}$$

$$\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + Pr\frac{\mu_{nf}/\mu_f}{\rho_{nf}/\rho_f}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \frac{\beta_{nf}}{\beta_f}RaPr \tag{4}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \alpha_{nf}/\alpha_f\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \tag{5}$$

$$\frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y} = -\nabla^2 \tag{6}$$

with boundary conditions

$$U = V = 0 \text{ at } \frac{1}{2} \leq X \leq 1, Y = -2X + 2 \tag{7}$$

$$U = V = 0 \text{ at } X = A \cdot (1 - \cos(\lambda Y)), 0 \leq Y \leq 1 \tag{8}$$

$$U = V = 0, \frac{\partial \theta}{\partial Y} = 0 \text{ at } Y = 0 \tag{9}$$

$$U = V = 0, \frac{\partial\theta}{\partial Y} = 0 \; and \; \frac{\partial U}{\partial Y} = Ma_{bf} \frac{\partial\theta}{\partial X} \text{ at } Y = 1 \tag{10}$$

$$\theta = 0 \text{ at } \frac{1}{2} \leq X \leq 1, Y = -2X + 2 \tag{11}$$

$$\theta = 1 \text{ at } X = A \cdot (1 - \cos(\lambda Y)), 0 \leq Y \leq 1 \tag{12}$$

using the following substitutions

$$X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{uL}{\alpha_f}, V = \frac{vL}{\alpha_f}$$

$$\theta = \frac{T - T_c}{T_h - T_c}, P = \frac{pL^2}{\rho_{nf}\alpha_f^2}, Pr = \frac{\mu_f}{\rho_f\alpha_f}$$

$$Ra = \frac{g\beta_f L^3(T_h - T_c)}{\upsilon_f\alpha_f}, Ma = -L\frac{\partial\sigma}{\partial T}\frac{(T_h - T_c)}{\mu_f\alpha_f}, \Psi = \frac{\psi}{\alpha_f}$$

Since nanofluid is being used in this study, the solid volume fraction of nanoparticles is denoted by $\phi$ and its density, heat capacitance, dynamic viscosity, thermal diffusivity, thermal expansion coefficient and thermal conductivity equations are given by

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s$$

$$(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s$$

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}$$

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}$$

$$\beta_{nf} = (1 - \phi)\beta_f + \phi\beta_s$$

$$k_{nf} = k_f \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f - \phi(k_f - k_s)}$$

The heat transfer of the heated wall is calculated by computing both the local and average Nusselt number where the two numbers give an idea of how much heat is being transferred over an area. The local Nusselt calculation at the hot wall is taken by considering Fourier's Law where

$$Nu_{loc} = \frac{k_{nf}}{k_f}\sqrt{\left(\frac{\partial\theta}{\partial X}\right)^2 + \left(\frac{\partial\theta}{\partial Y}\right)^2} \tag{13}$$

The average Nusselt is then computed by taking the integral of the equation above to get

$$Nu_{avg} = \int_0^1 Nu_{loc}\frac{\partial\theta}{\partial N}dN \tag{14}$$

**Table 1:** Thermo-physical properties of water and aluminium oxide

| Physical Properties | Water | Aluminium Oxide |
|---|---|---|
| $\rho$ | 997.1 | 3600 |
| $C_p$ | 4179 | 765 |
| $\kappa$ | 0.605 | 46 |
| $\beta \times 10^{-5}$ | 21 | 0.63 |

*2.1 Numerical Technique*

In order to numerically solve the governing equations with respect to the boundary conditions, COMSOL 5.3a was used. The software utilizes the finite element method [FEM] where it is described in detail by Taylor and Hood, Nasrin and Parvin and Dechumpai.

## 3. Results and discussion

A numerical study was carried out to investigate Marangoni convection in a wavy trapezoidal enclosure with different amplitudes. The heated and cooled walls were set to fixed temperatures. In this paper, the Prandtl number, $Pr$ was set to \$ 0.71 \$ for all numerical computations. The streamlines, isotherms, local Nusselt and average Nusselt were presented for various values of $Ma$, $Ra$, $A$ and $\phi$ with $0 \leq Ma \leq 10^3$, $10^3 \leq Ra \leq 10^4$, $0.1 \leq A \leq 0.2$ and $0 \leq \phi \leq 0.03$. Figure 2 - 7 show the fluid flow circulation inside the enclosure for various values of $Ma$, $Ra$ and $A$. For all $\lambda$, we see that when the amplitude is increased from 0.1 to 0.2, the fluid flow weakens in general. It can also be observed that the weaker of the two cells that form as \$ Ma \$ is increased, becomes smaller as $A$ is increased, suggesting that while $Ma$ does help strengthen the flow of the vortices, a bigger value of $A$ has the opposite effect.

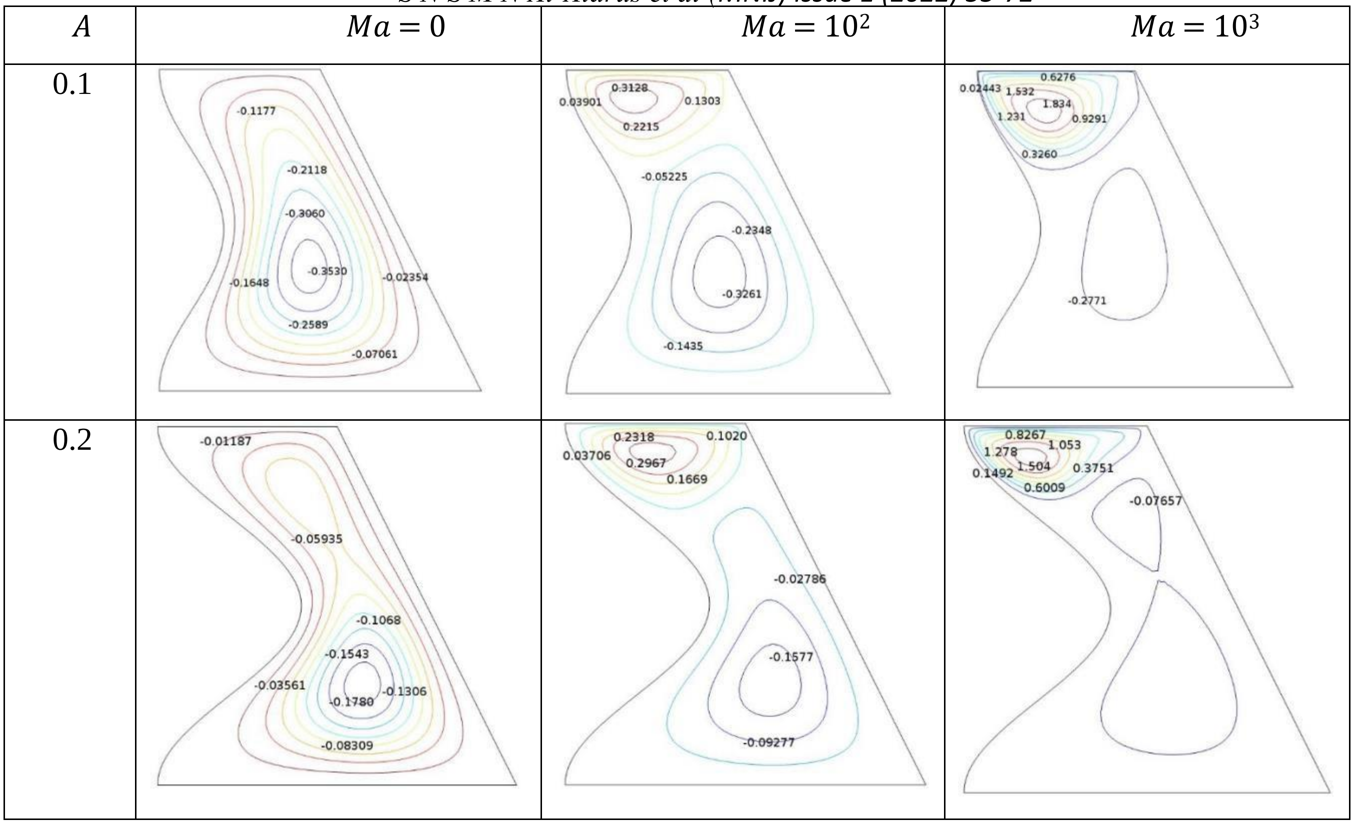



**Figure 2:** Streamlines for $Ra = 10^3$ and $\lambda = 1$

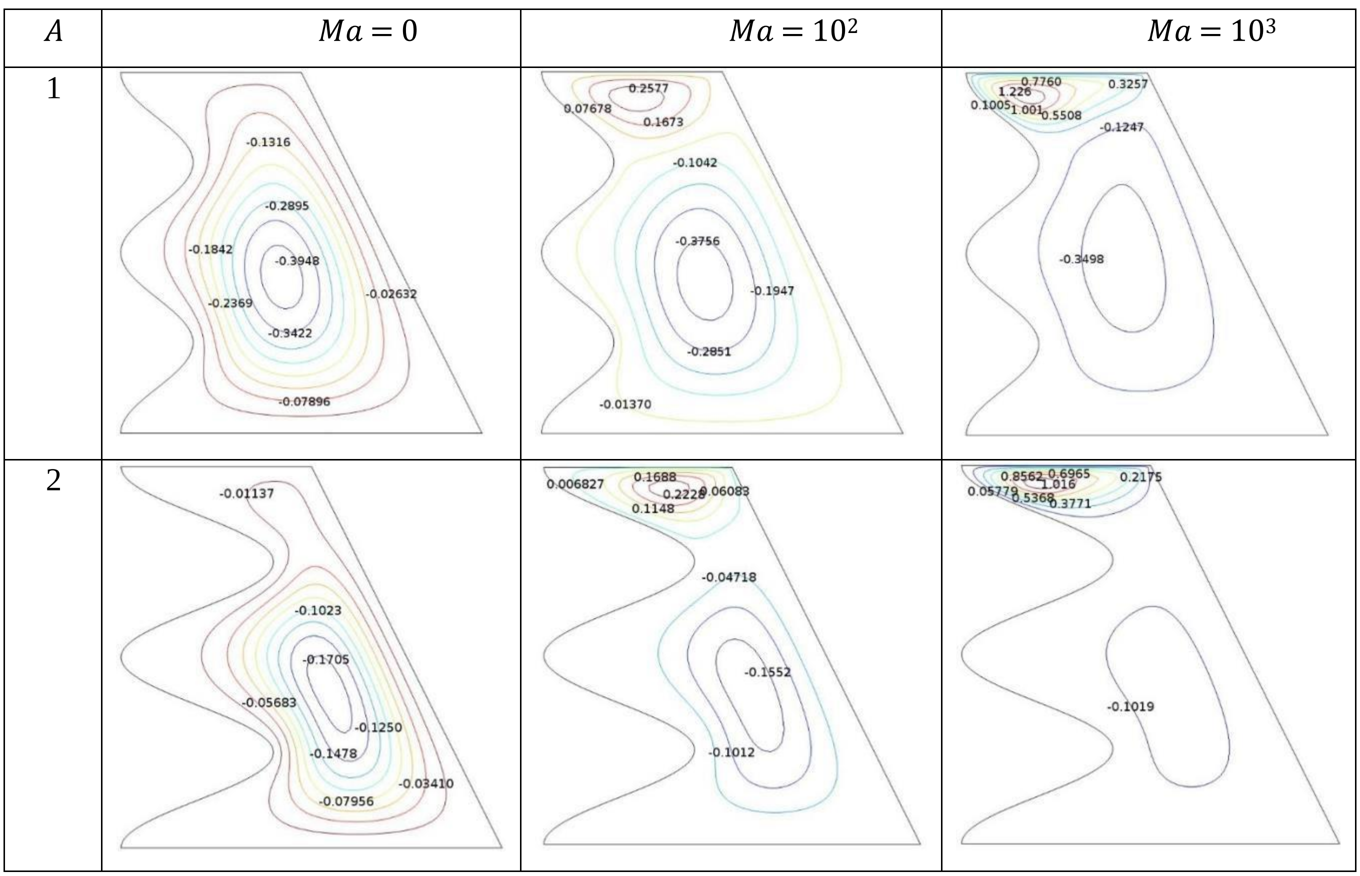



**Figure 3:** Streamlines for $Ra = 10^3$ and $\lambda = 2$

Figure 8 - 13 illustrate the isotherms for $Ra$ and $Ma$ with $A = 0.1$ and $A = 0.2$. In general, it can be seen that there is little to no difference in the the temperature contours. The most significant distortions are visible in the top part of the enclosure with $\lambda = 1$ when $Ma > 0$. The isotherms also curve better to the grooves of the wavy wall for $\lambda = 1$ and $\lambda = 2$, regardless of the value of the amplitude, suggesting that $A$ does not affect heat flow all that much. Figure 14 depicts the local Nusselt number of the hot wall for various wavelengths and amplitude with varying Ma and Ra. All graphs are periodic due to the shape of the wall.

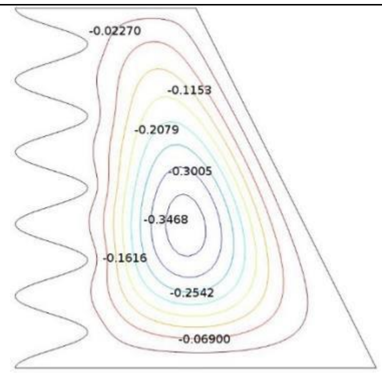
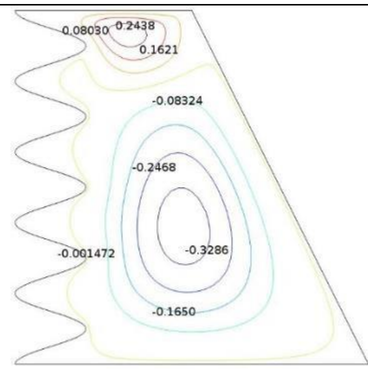
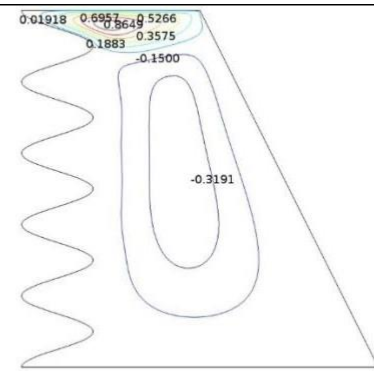
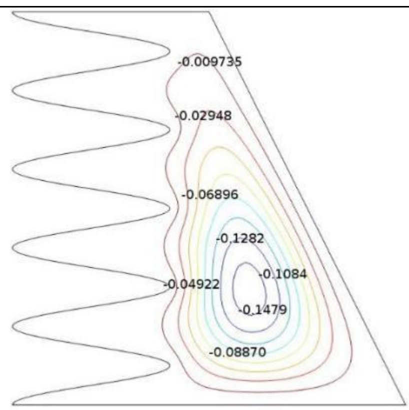
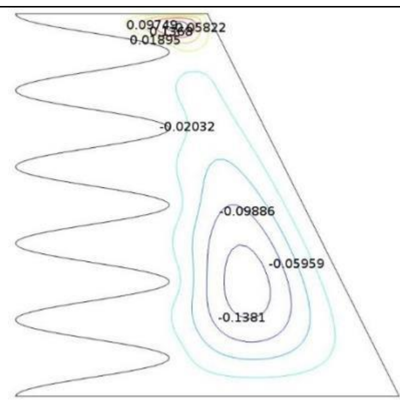
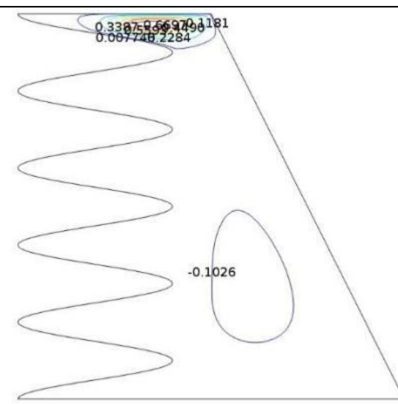
| $A$ | $Ma = 0$ | $Ma = 10^2$ | $Ma = 10^3$ |
|---|---|---|---|
| 0.1 |  |  |  |
| 0.2 |  |  |  |

**Figure 4:** Streamlines for $Ra = 10^3$ and $\lambda = 5$

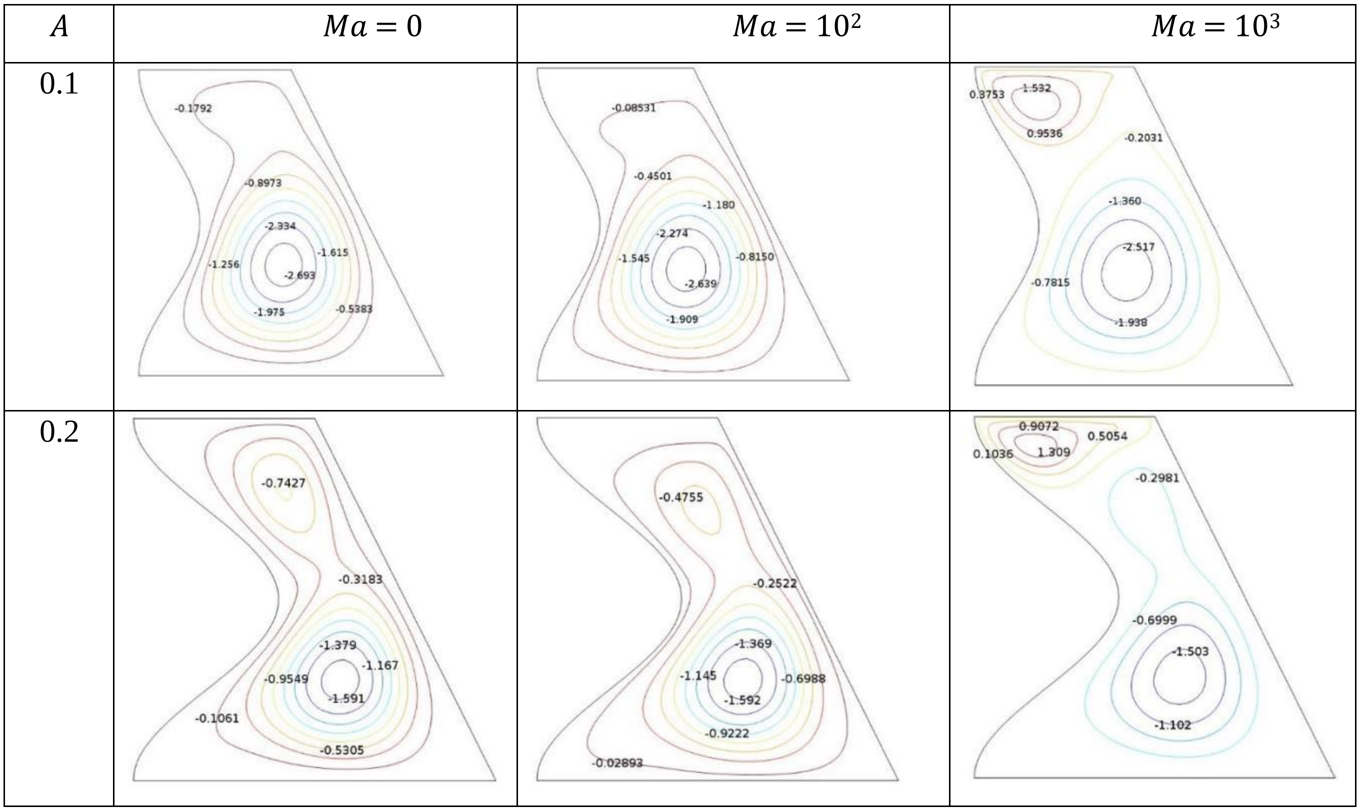



**Figure 5:** Streamlines for $Ra = 10^4$ and $\lambda = 1$

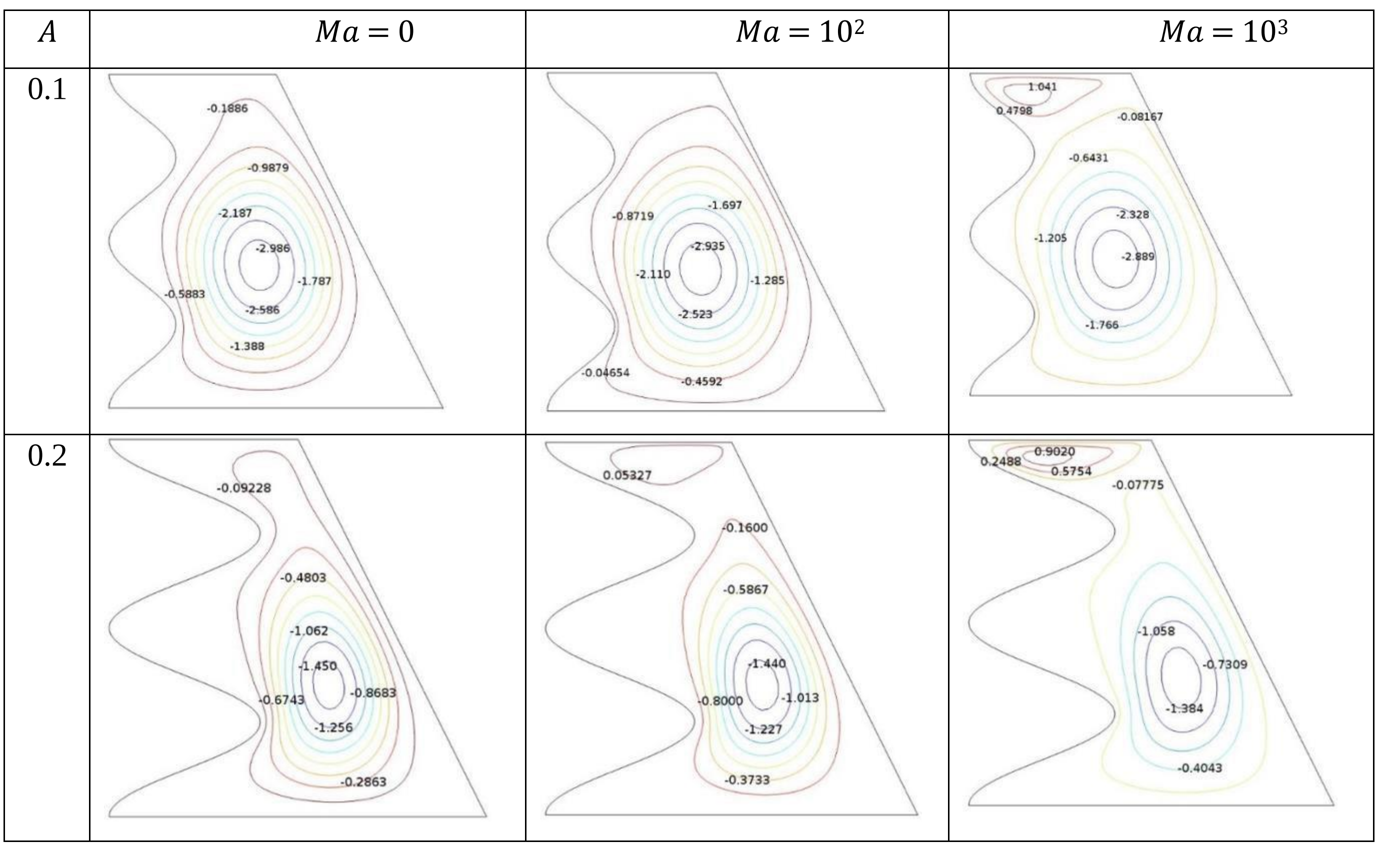



**Figure 6:** Streamlines for $Ra = 10^4$ and $\lambda = 2$

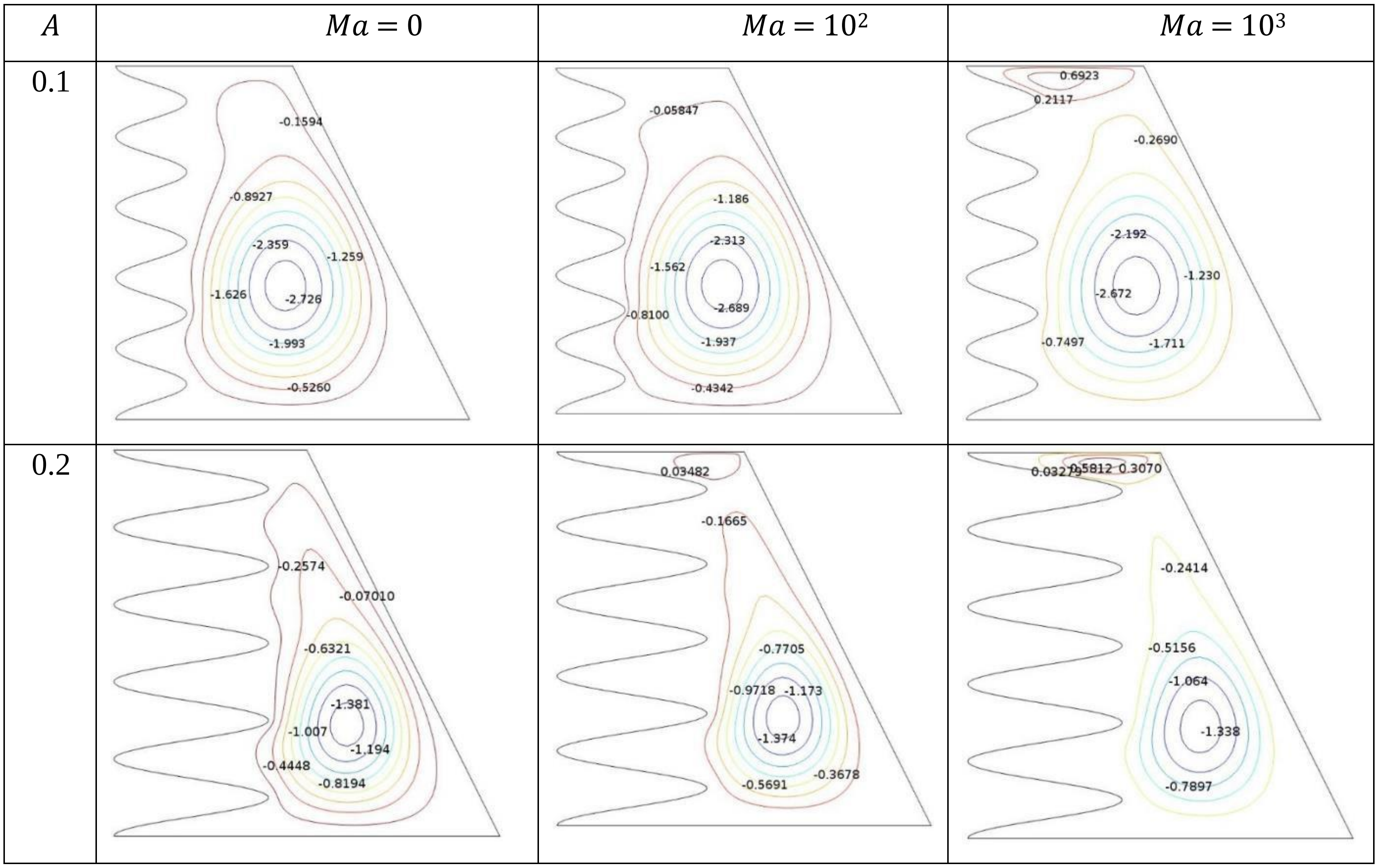



**Figure 7:** Streamlines for $Ra = 10^4$ and $\lambda = 5$

From the figure, it can be observed that highest heat transfer rates for the hot wall are achieved when $\lambda = 5$ and $A = 0.2$ for all $Ma$ and $Ra$. However, the local Nusselt number for $\lambda = 1$ and $\lambda = 2$ only increased slightly when $A = 0.2$ compared to $A = 0.1$. Note that when the Rayleigh number is increased to $10^4$, $Nu_{loc}$ takes a slight dip for all $Ma$, $\lambda$ and $A$. The graphs for $\lambda = 1$ also shift slightly to the left and right, depending on the amplitude. The peak value of $Nu_{loc}$ for $\lambda = 5$ is located at $Y = 0.9$, close to where Marangoni convection takes place, implying that natural convection helps with heat transfer.

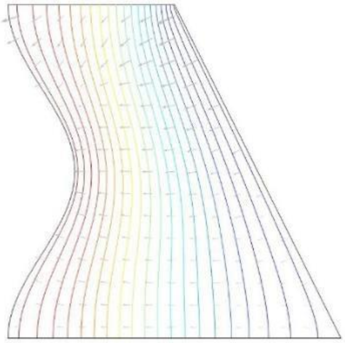
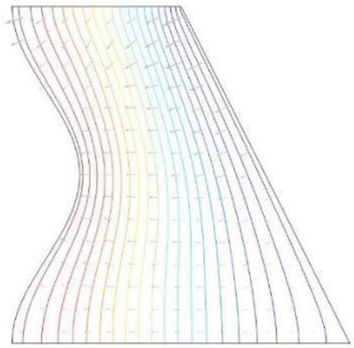
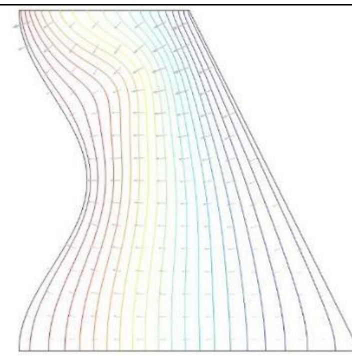
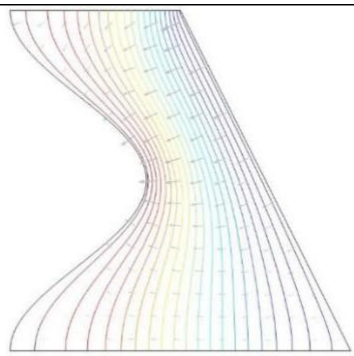
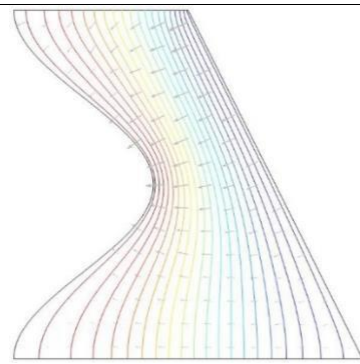
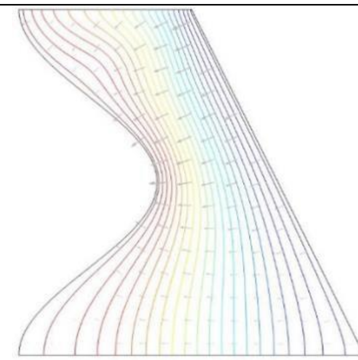
| $A$ | $Ma = 0$ | $Ma = 10^2$ | $Ma = 10^3$ |
|---|---|---|---|
| 0.1 |  |  |  |
| 0.2 |  |  |  |

**Figure 8:** Isotherms for $Ra = 10^3$ and $\lambda = 1$

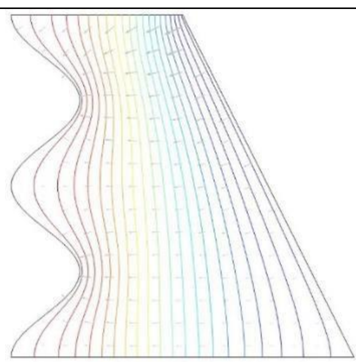
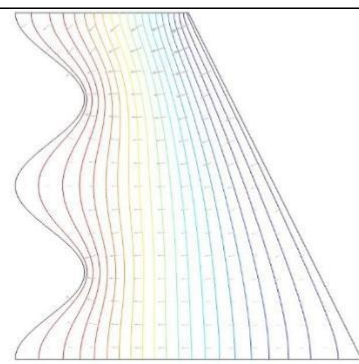
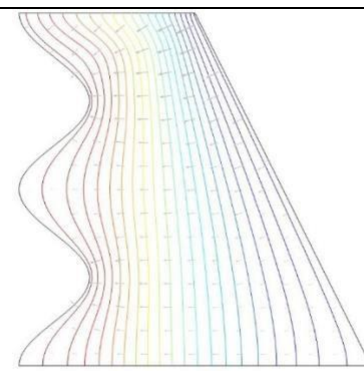
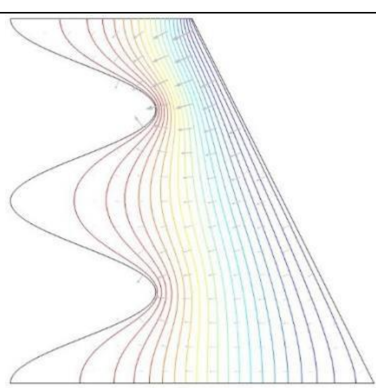
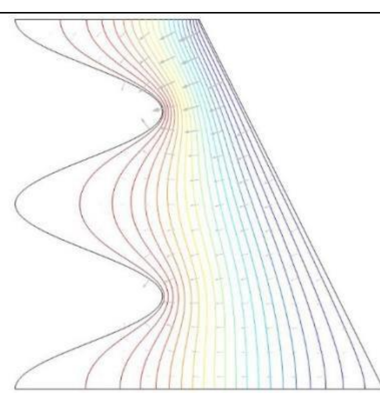
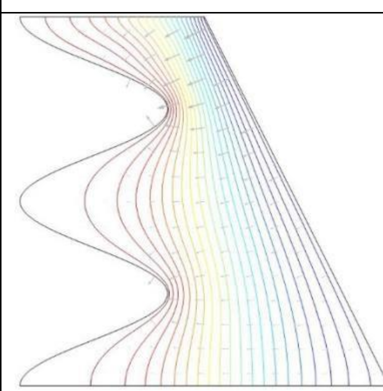
| $A$ | $Ma = 0$ | $Ma = 10^2$ | $Ma = 10^3$ |
|---|---|---|---|
| 0.1 |  |  |  |
| 0.2 |  |  |  |

**Figure 9:** Isotherms for $Ra = 10^3$ and $\lambda = 2$

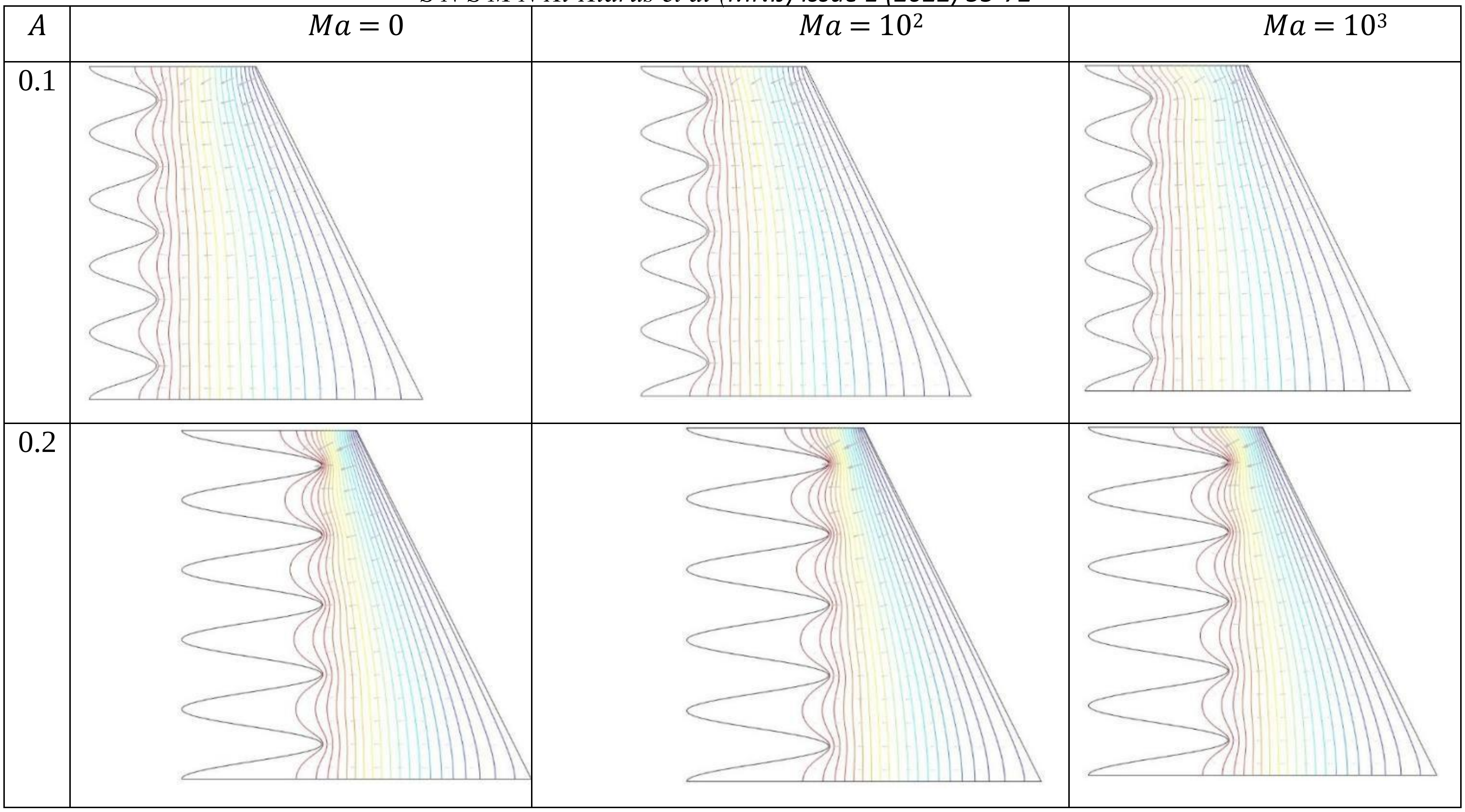



**Figure 10:** Isotherms for $Ra = 10^3$ and $\lambda = 5$

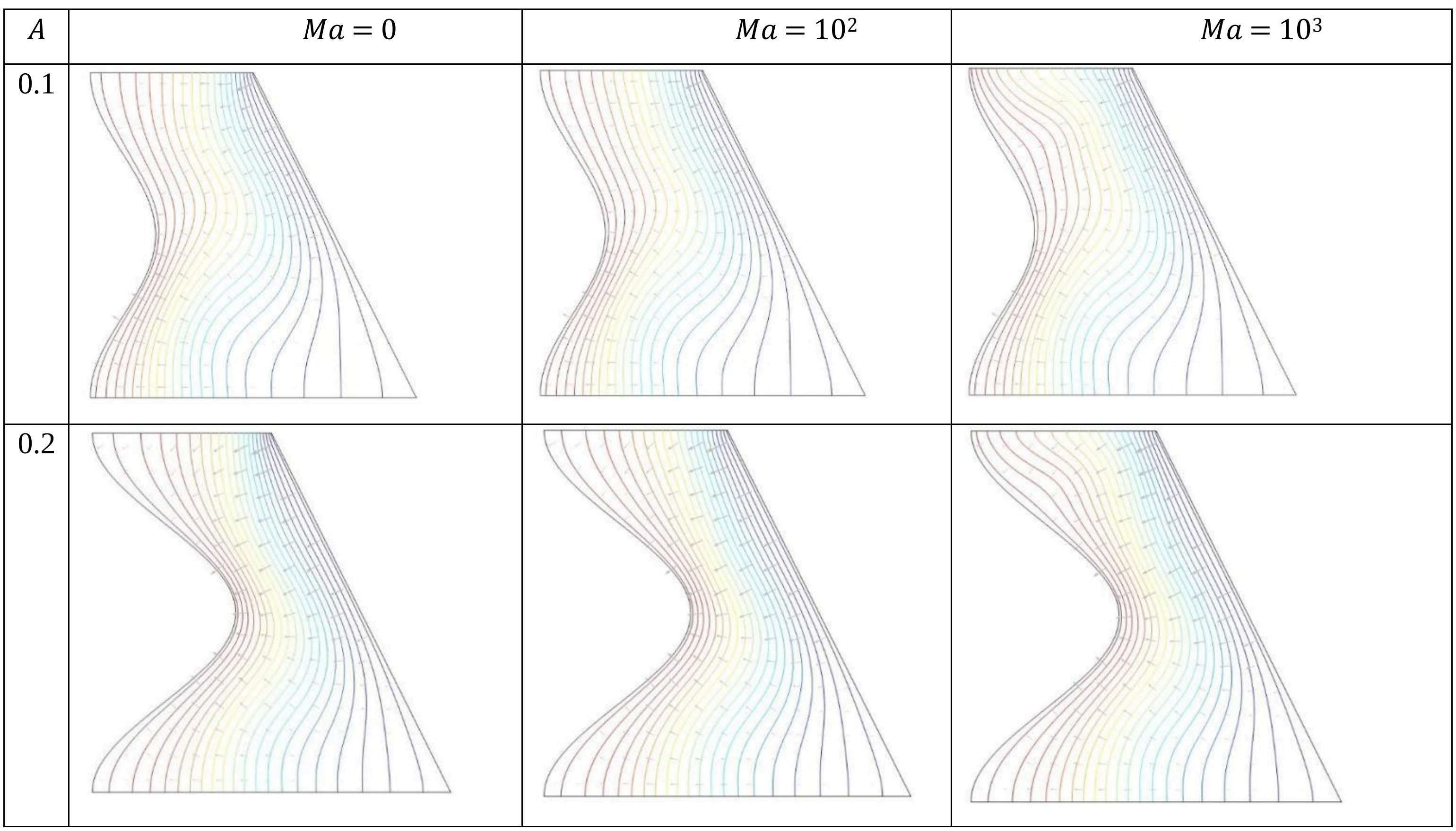



**Figure 11:** Isotherms for $Ra = 10^4$ and $\lambda = 1$

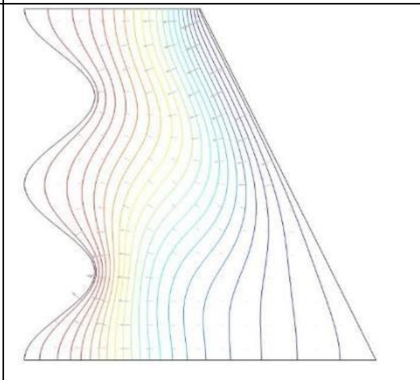
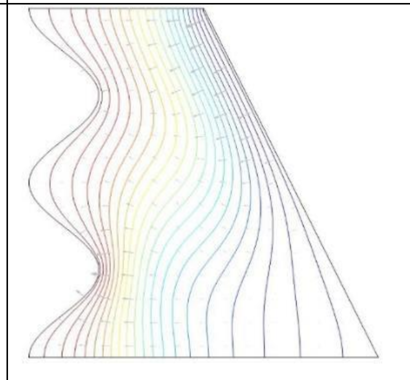
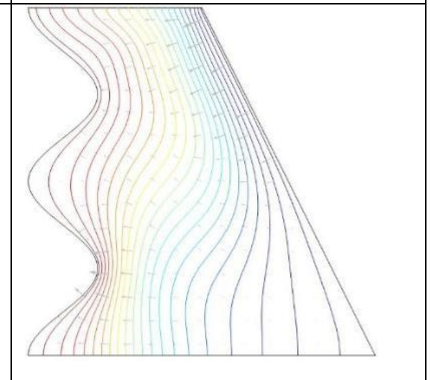
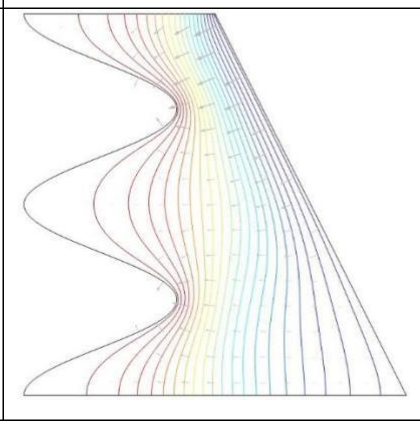
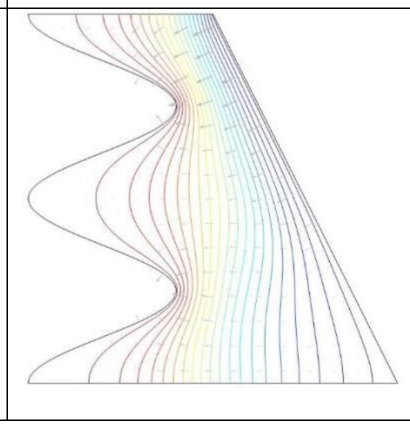
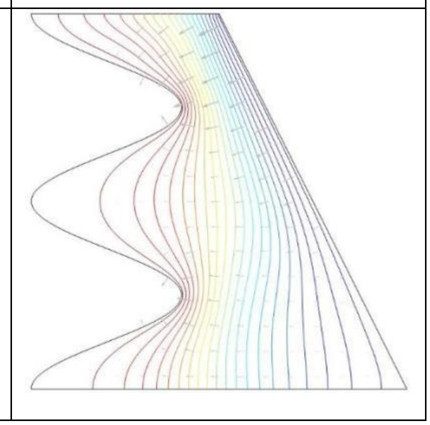
| $A$ | $Ma = 0$ | $Ma = 10^2$ | $Ma = 10^3$ |
|---|---|---|---|
| 0.1 |  |  |  |
| 0.2 |  |  |  |

**Figure 12:** Isotherms for $Ra = 10^4$ and $\lambda = 2$

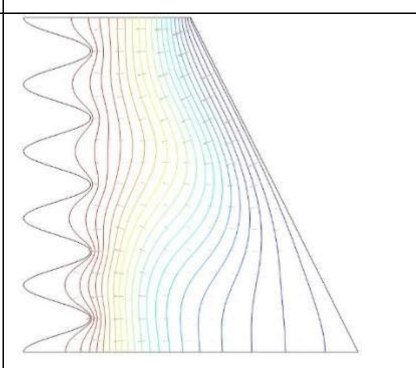
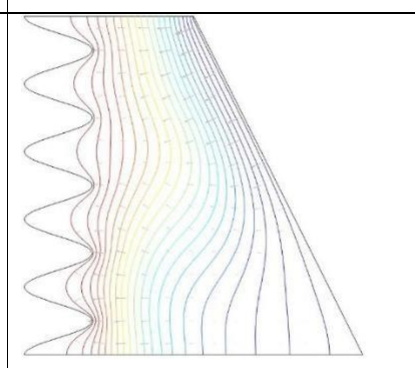
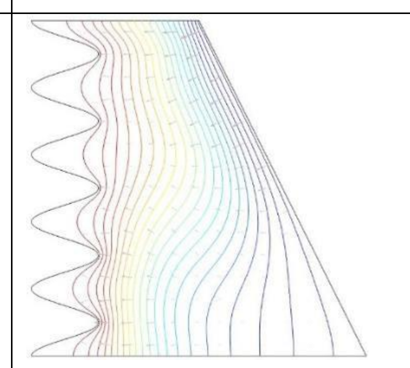
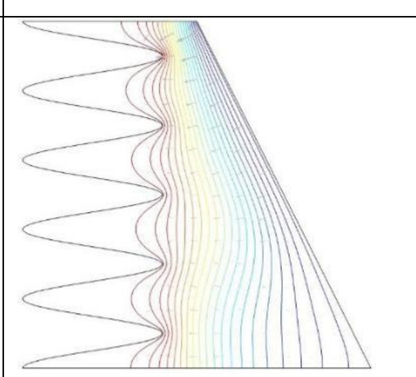
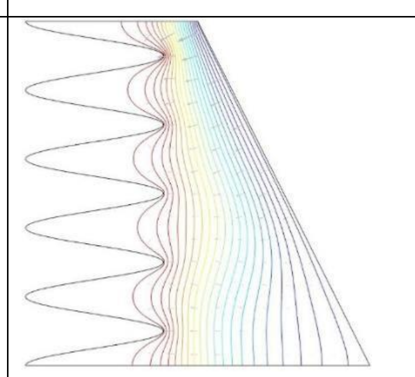
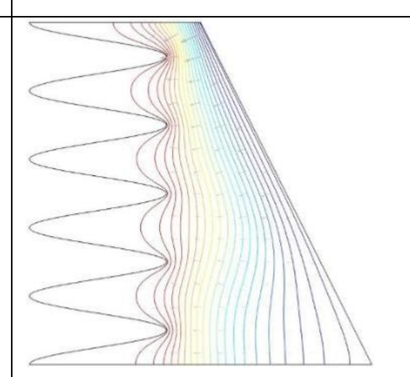
| $A$ | $Ma = 0$ | $Ma = 10^2$ | $Ma = 10^3$ |
|---|---|---|---|
| 0.1 |  |  |  |
| 0.2 |  |  |  |

**Figure 13:** Isotherms for $Ra = 10^4$ and $\lambda = 5$

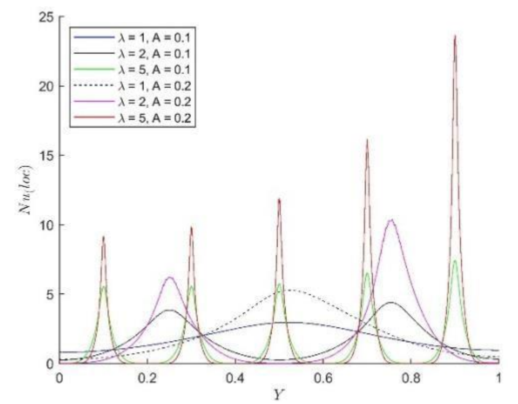
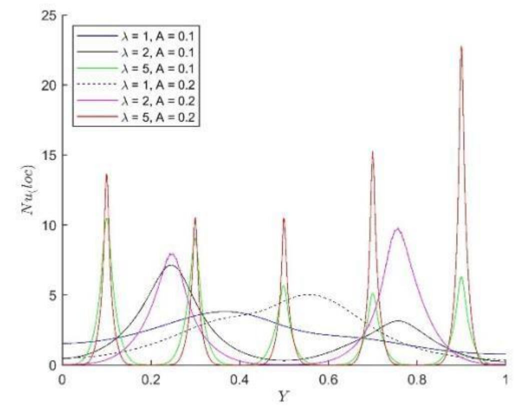
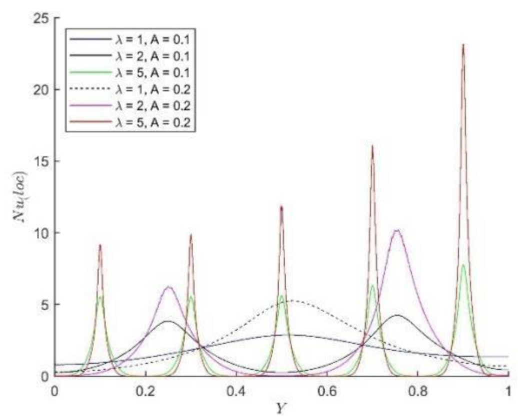
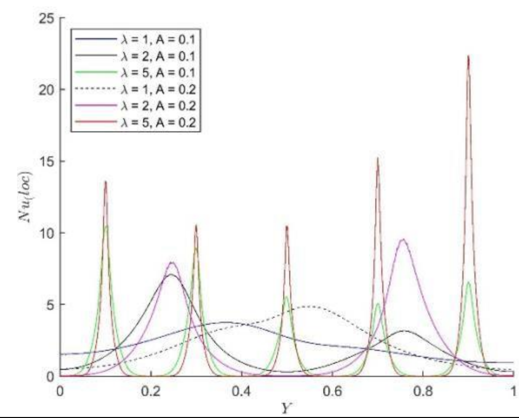
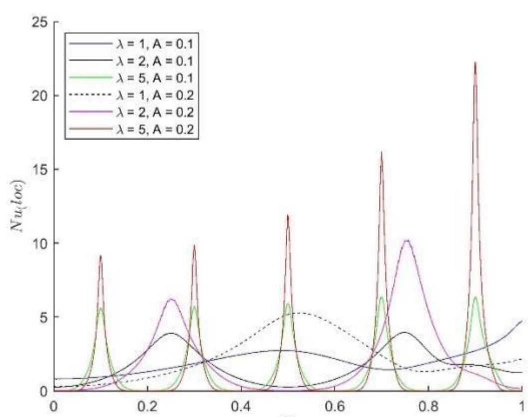
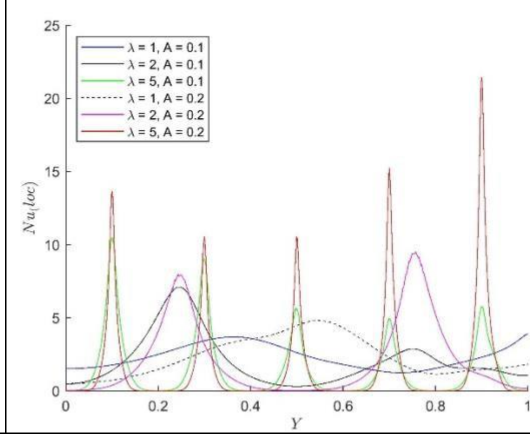
| $Ma$ | $Ra = 10^3$ | $Ra = 10^4$ |
|---|---|---|
| 0 |  |  |
| $10^2$ |  |  |
| $10^3$ |  |  |

**Figure 14:** Local Nusselt for various values of $Ra$, $Ma$ and λ

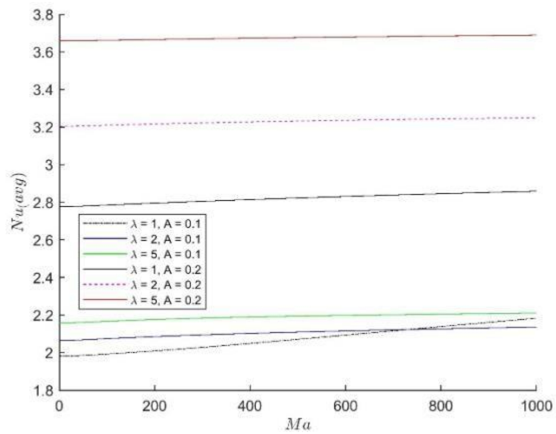
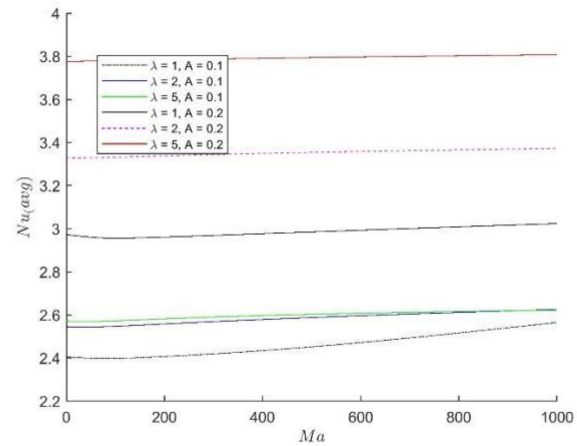
<table>
<tr><td>$Ra$</td><td>$10^3$</td><td>$10^4$</td></tr>
<tr><td></td><td>
</td><td>
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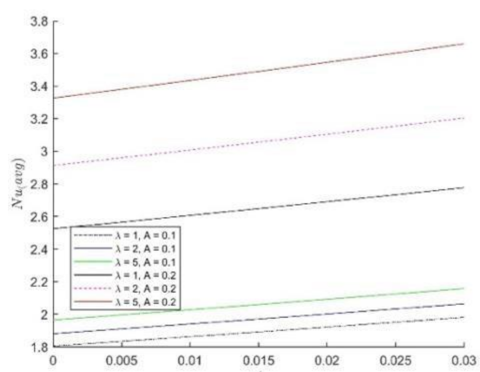
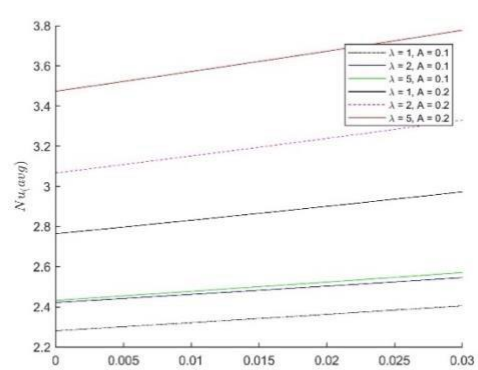
**Figure 15:** Average Nusselt for various values of $Ra$ and λ with respect to $Ma$

Increasing the Marangoni number also improves the heat transfer rate, though not by much. Figure 15 depicts the average Nusselt number with respect to $Ma$ for various amplitudes and $Ra$. $Nu_{avg}$ increased linearly with $Ma$ and the highest values are obtained for $\lambda = 5$ with amplitude $A = 0.2$. In addition to that, when $Ma$ is big enough, specifically around the 700 mark, the enclosure with $\lambda = 1$ and $A = 0.1$ has higher $Nu_{avg}$ compared to when the amplitude was $\lambda = 2$ for $Ra = 10^3$. Figure 16 illustrates the $Nu_{avg}$ with respect to ϕ and similarly to the previous figure, the Nusselt number for all cases increased linearly for all $Ma$ and $Ra$ with the highest heat transfer rates achieved by the cavity with $\lambda = 5$ and $A = 0.2$. From both the local and average Nusselt figures, increasing the amplitude significantly helps with heat transfer.

<table>
<tr><td>$Ma$</td><td>$Ra = 10^3$</td><td>$Ra = 10^4$</td></tr>
<tr><td>0</td><td>
</td><td>
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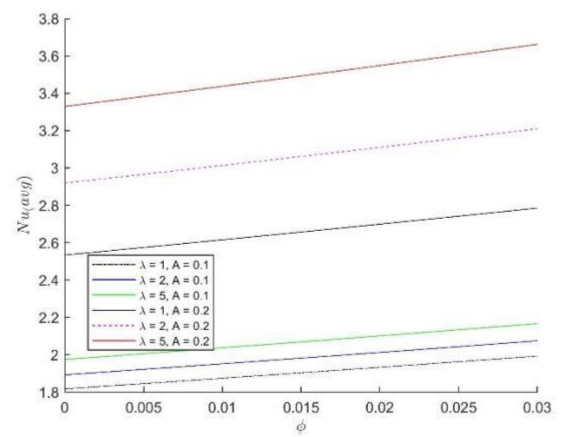
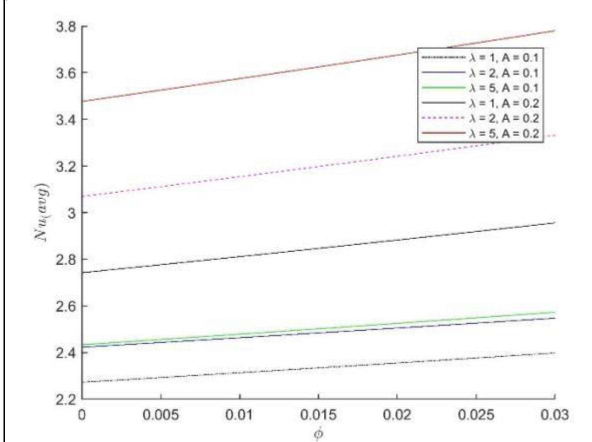
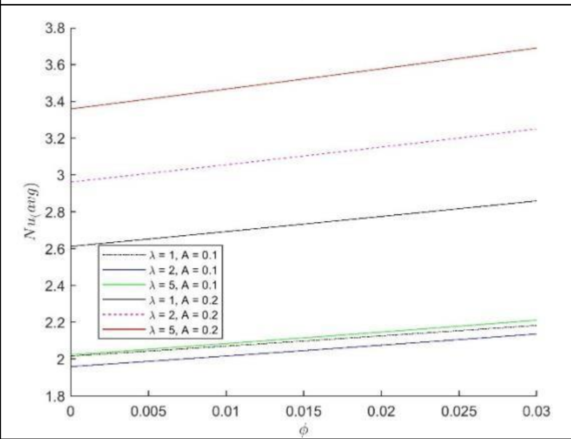
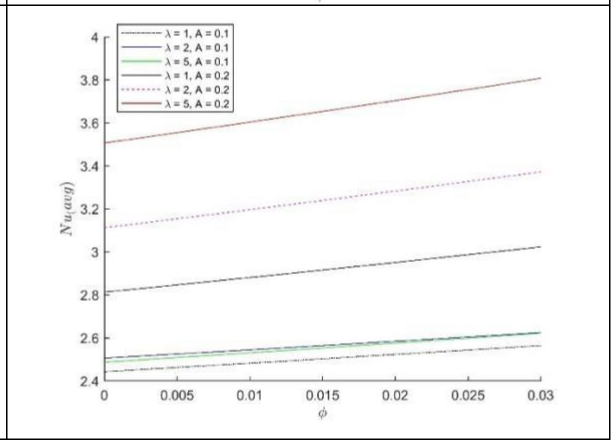
| | | |
|---|---|---|
| $10^2$ |  |  |
| $10^3$ |  |  |

**Figure 16:** Average Nusselt for various values of Ma, Ra and lambda with respect to \phi

## 4. Conclusions

A numerical study was conducted to investigate Marangoni convection in a trapezoidal enclosure with a heated wavy wall for different amplitudes where two cases of A were considered: $A = 0.1$ and $A = 0.2$. The governing equations and boundary conditions of the mathematical problem were then non-dimensionalised before solved numerically using COMSOL 5.3a. The results were in the form of streamlines, isotherms, local Nusselt and average Nusselt for various values of $Ra, Ma,$ and ϕ. It can be concluded that:

- The amplitude helps with heat transfer as increasing $A$ increase the local and average Nusselt number.
- Increasing the amplitude slows down fluid flow slightly.

**Acknowledgements**

The work is supported financially by the Ministry of Higher Education Malaysia via Fundamental Research Grant Scheme (FRGS/1/2020/STG06/UM/02/4) and Impact Oriented Interdisciplinary Research Grant No. IIRG006C-19IISS.

**Conflicts of Interest**

The authors declare no conflict of interest.

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