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Forecasting GDP Growth of Brunei Darussalam Using Factor Models

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Abstract: This paper evaluates the relative performance of factor models in forecasting GDP growth using a large quarterly panel dataset compiled for the Brunei economy. The common factors are extracted through the estimation of both static and dynamic principal components, and are used to compute pseudo out-of-sample forecasts in a recursive scheme. These factor-based forecasts are then compared to a standard benchmark univariate autoregressive model. The forecasting results show that the forecast errors of the benchmark model increase with the prediction horizon but the forecast errors of factor models remain relatively unchanged. In spite of poorer forecasting performance in one- and two-quarter ahead forecasts, factor models significantly outperform the benchmark in three- and four-quarter ahead forecasts. This implies that the information conveyed by the large dataset provides predictive power at longer horizons, illustrating the usefulness of factor models as a macroeconomic forecasting tool for Brunei.

Keywords: Brunei Darussalam, factor models, GDP forecasting, principal components
JEL classification: C32, E37

1. Introduction

Economic decisions, whether they are made by policymakers, businesses or consumers, rely on accurate forecasts of key macroeconomic variables such as output and inflation. However, due to the lack of economic modelling and forecasting tools in Brunei Darussalam, the central bank and government agencies do not produce any forecasts on the economic outlook. The only publicly available forecasts for Brunei are from the International Monetary Fund (IMF), Asian Development Bank (ADB) and the Organisation for Economic Co-operation and Development (OECD), published on an annual basis. The use and level of sophistication in economic modelling techniques in Brunei is at an infancy stage; it is only recently that the Department of Economic Planning and Development (DEPD) constructed the first Input-Output table and developed a Computable General Equilibrium (CGE) model to aid in policy analysis (see Masli and Low 2012). There are also plans to develop structural macroeconometric models for forecasting purposes. However, traditional economic models such as simultaneous equation systems or vector autoregressions (VAR) cannot accommodate a large number of variables without running short of degrees of freedom. Econometricians therefore rely on parsimonious specifications which leave an enormous amount of information unused.

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Central banks closely monitor hundreds of economic indicators from various sources, which Bernanke and Boivin (2003) call "looking at everything". Following the work of Stock and Watson (2002a; 2002b), the use of factor models in forecasting macroeconomic variables in a data-rich environment has become increasingly popular over the past decade. Factor models allow circumventing the curse of dimensionality by compressing the information conveyed by a plethora of indicators into a few unobserved common factors, which are interpreted as the forces driving the economy. Although factor models have been criticised as an agnostic approach in modelling economic dynamics, the improved forecast accuracy from these models suggests that they are a useful toolkit for policymakers that complements other large-scale macroeconomic models.

Most of the studies in the literature focus on the advanced and emerging economies (see Eickmeier and Ziegler 2006 for a list of 46 studies). While the empirical literature suggests that factor-based forecasts usually outperform benchmark models such as univariate models and VARs, the benefits are not always statistically significant. According to Boivin and Ng (2006), the composition of the dataset and the size of the cross-section dimension are important in producing accurate forecasts. Therefore factor model forecasts should not be uncritically embraced without prior empirical evaluation, especially for small developing countries with different economic mechanisms.

Unlike the more advanced economies, many indicators that are potentially useful in predicting output of Brunei such as business confidence surveys are not collected, and labour market variables are unavailable at the quarterly and monthly frequency. Moreover, Brunei is vulnerable to external shocks such as terms of trade shocks due to its high dependence on oil and gas, as well as foreign monetary policy shocks as it adopts a currency peg to the Singapore dollar and therefore imports Singapore's monetary policy. Therefore, oil market shocks and monetary policy spillovers can have important effects on Brunei's economy.

Time series data in Brunei are also relatively short; for example, quarterly GDP is only available starting 2003Q1. This hinders the use of traditional parametric econometric models due to less efficient estimates, and factor models are therefore advantageous. The purpose of this paper is to assess whether factor models are useful in producing accurate forecasts of GDP growth in Brunei using a constructed quarterly panel dataset of 107 macroeconomic indicator variables.

The rest of this paper is organised as follows. Section 2 provides a brief overview of the two popular factor models used in forecasting macroeconomic variables. Section 3 describes the dataset and the necessary data transformations. Section 4 discusses the forecasting strategy, out-of-sample forecast evaluation methods and the forecasting results. Section 5 summarises and concludes.

2. Forecasting Using Factor Models

Factor models summarise the information contained in a large cross-section of time series into a small number of common factors or common shocks. The basic underlying idea is that the movement of a time series can be characterised as the sum of two mutually orthogonal unobservable components: a common component and an idiosyncratic component. The common component is a linear combination of the common shocks, is strongly correlated with the rest of the panel, and explains a large proportion of the variance of the time series. In contrast, the idiosyncratic component is a variable-specific shock and is weakly correlated across the panel. This idea is implicit in the seminal work of Burns and Mitchell (1946) in their analysis of business cycles through an index model using a static factor methodology. Sargent and Sims (1977) and Geweke (1977) subsequently generalise to the dynamic case by exploiting the dynamic interrelationship of the variables and further reducing the number of common factors. However, their approach is too restrictive as it imposes orthogonality of idiosyncratic components. The approximate factor models of Chamberlain (1983) and Chamberlain and Rothschild (1983) allow heteroscedasticity and weak serial and cross-correlation between idiosyncratic components.

These factor models have been improved over time through advances in estimation techniques by Stock and Watson (2002a; 2002b) and Forni *et al.* (2000; 2004; 2005). Both the Stock and Watson (SW henceforth) and the Forni *et al.* (FHLR henceforth) approaches combine the approximate factor model and the dynamic factor model. However, there are two main differences between the SW and FHLR methodologies. First, the unobservable common and idiosyncratic components are estimated using static principal components based on an eigenvalue decomposition of the contemporaneous covariance matrix in the SW time domain approach, while dynamic principal components are used based on the spectral density matrix (i.e. dynamic covariations) of the data in the FHLR frequency domain approach. Second, SW exploit the factor structure only in the estimation stage whereas FHLR exploit the factor structure in both the estimation and forecasting procedures. It is not clear whether the generalisation of FHLR is superior to the SW model (see Boivin and Ng 2005). For example, if the data generating process comes from a simple static factor model, then the FHLR method uses too many covariance estimates leading to a loss of efficiency. On the other hand, if there exists dynamic correlation between the predictors, the SW model could be misspecified.

This section provides a brief overview on the estimation and forecasting procedures based on these two approaches. Further technical details are in the SW and FHLR papers.

2.1 Model of Stock and Watson

Following Stock and Watson (2002b), let y_t be a zero-mean scalar time series to be forecast and X_t be an *N*-dimensional stationary zero-mean vector of potential predictors of y_t for a horizon of up to *h*-steps ahead, with $t = 1, ..., T$ and $h = 1, ..., H$. Assume that the joint process (y_t, X_t) has the following factor model representation:

$$y_{t+h} = \beta(L)f_t + \gamma(L)y_t + \epsilon_{t+h} \tag{1}$$

$$X_{it} = \lambda_i(L)f_t + \xi_{it} \tag{2}$$

for $i = 1, ..., N$, where ε_{t+h} is the *h*-step ahead error, ξ_{it} are idiosyncratic component errors, f_t is a vector of common *q* dynamic factors, and $\beta(L)$, $\gamma(L)$ and $\lambda_i(L)$ are lag polynomials.

If the lag polynomials are assumed to have a finite order of at most *s*, (1) and (2) can be rewritten in a static form as:

$$y_{t+h} = \beta' F_t + \gamma(L)y_t + \epsilon_{t+h} \tag{3}$$

$$X_t = \Lambda F_t + \xi_t \tag{4}$$

where $F_t = (f'_t,..., f'_{t-s})$ is the r-dimensional vector with $r \leq q$ (s+1), Λ is the matrix of the coefficients λ_i where the i-th row is $(\lambda_{i0},...,\lambda_{is})$ and $\beta = (\beta_0,...,\beta s)'$. q is the number of dynamic factors (f_t) while r represents the number of static factors (F_t).

In this static representation, it is easy to estimate the model parameters using principal components (PC). The PC estimator is derived as the solution to the least squares problem:

$$min_{F_1,...,F_T} V_t(\Lambda, F) = \frac{1}{NT}\sum_{t=1}^{T}(X_t - \Lambda F_t)'(X_t - \Lambda F_t)$$

subject to $N^{-1}\Lambda'\Lambda = I_r$. Solving this minimisation problem gives the estimates of the factor loadings and the factors. The estimates of the factor loadings, $\widehat{\Lambda}$, are the eigenvectors corresponding to the r largest eigenvalues of matrix $X'X$ (arranged in descending order), and the factor estimates, $\hat{F}$, are $\hat{F} = \frac{X'\widehat{\Lambda}}{N}$.

The conditional h-step ahead forecasts can then be constructed as:

$$\hat{y}_{t+h|t} = \hat{\beta}(L)'\hat{F}_t + \hat{\gamma}(L)y_t \tag{5}$$

where the regression coefficients $\hat{\beta}(L)$ and $\hat{\gamma}(L)$ are estimated using least squares. Note that forecasting is conducted using direct projection on data available until time t, with the assumption that $E(\epsilon_{t+h}|I_t) = 0$ where I_t is the information available up to time t.

2.2 Model of Forni, Hallin, Lippi and Riechlin

The generalised dynamic factor model (GDFM) proposed by Forni *et al.* (2000, 2005) is presented here. The representation theory is elaborated in Forni and Lippi (2001). Consider a zero-mean stationary N-dimensional vector process $\chi_{it} = (\chi_{1t}, ..., \chi_{nt})'$. Under the GDFM, satisfying the necessary conditions and assumptions, each variable x_{it} can be decomposed into two components: the common component χ_{it} and the idiosyncratic component ξ_{it}. The common component is driven by q-dimensional vector of common factors. These factors are the same for all variables but are loaded with different coefficients and lag structure. That is,

$$X_{it} = \chi_{it} + \xi_{it} = b_i(L)u_t + \xi_{it} = \sum_{j=1}^{q} b_{ij}(L)u_{jt} + \xi_{it} \tag{6}$$

where $b_i(L)= (b_{i1}(L), ... b_{iq}(L))$ is a vector of lag polynomials and $u_t=(u_{1t},...,u_{qt})$ is the vector of common shocks assumed to be mutually orthogonal white noise processes at all leads and lags, with unit variance.

In matrix notation, Equation (6) can be rewritten as:

$$X_t = \chi_t + \xi_t = B(L)u_t + \xi_t = \Lambda F_t + \xi_t \tag{7}$$

where $B(L) = B_0 + B_1L + ... + B_sL^s$ is a N x q polynomial matrix of order s in the lag operator L. $X_t = B(L)u_t + \xi_t$ is the dynamic factor model and $X_t = \Lambda F_t + \xi_t$ is the static form where F_t is a r-dimensional vector of factors.

The model is general since it does not impose restrictions on the order of the dynamic loadings of the common factor, and the idiosyncratic component is allowed to be mildly cross-correlated at all leads and lags. However, assuming away the orthogonality conditions between the idiosyncratic components requires assumptions on the eigenvalues of the spectral density matrix of the data to separate the idiosyncratic sources of variation from the common ones to identify the model. In the GDFM, it is required that the first q eigenvalues of the spectral density matrix diverge, while the others remain bounded. This is to ensure that the shocks are present in infinitely many cross-sectional units so that there is a non-decreasing contribution to the variance of a progressively larger panel. This divergence assumption also ensures a minimum amount of correlation between the common components. The assumption on being bound ensures that the variance explained by the idiosyncratic components tend to zero as $N \to \infty$.

In contrast to the static PC estimation in the SW model, the common factors in the FHLR model are estimated using dynamic PC. While the static PC are only based on the contemporaneous covariances, in the dynamic PC, the data are shifted through time before averaging along the cross-section, accounting for the whole set of dynamic covariances. The common components are the orthogonal projections of the data on the present, past and future of the first q dynamic PCs, whereas the idiosyncratic components are the projections on the remaining $N - q$ dynamic PCs.

The GDFM relies on the spectral density matrix of the data, which are decomposed into common and idiosyncratic components by a dynamic PC decomposition for each frequency $-\pi < \theta < \pi$:

$$\Sigma(\theta) = \Sigma_{\chi}(\theta) + \Sigma_{\xi}(\theta) \tag{8}$$

where $\Sigma_{\chi}(\theta)$ is the spectral density matrix of the common component χ_t and $\Sigma_{\xi}(\theta)$ is the spectral density matrix of the idiosyncratic component ξ_t. The rank of $\Sigma_{\chi}(\theta)$ is equal to the number of dynamic factors, q.

Similarly, the covariance matrix of χ_t can be decomposed into:

$$\Gamma_k = \Gamma_k^{\chi} + \Gamma_k^{\xi} \tag{9}$$

where $\Gamma_k^{\chi} = \Lambda\Gamma_k^{F}\Lambda'$ is the covariance of χ_t, Γ_k^{F} is the covariance of F_t at lag k, and Γ_k^{ξ} is the covariance of ξ_t. The rank of Γ_k^{χ} is equal to the number of static factors, r.

The projection coefficients of the common components, $b_{ij}(L)$, are obtained from an inverse Fourier transform of the first q dynamic eigenvectors. An unpleasant feature of this estimator is that it is based on a two-sided filter, where both lagged and future values of the common shocks can be loaded. This leads to poor forecasting performance as t approaches either T or 1. Therefore the common components are poorly estimated at the end of sample since no future observations are available. Forni *et al.* (2005) propose an efficient procedure based on a one-sided filter of the observations to solve this problem. The procedure consists of two steps. First, the covariance matrices of the common and idiosyncratic components are derived through an inverse Fourier transformation of the spectral density matrices. Second, the estimated covariance matrix of the common component is used to construct the factor space by r contemporaneous averages. These r aggregates are the solutions from a generalised PC problem. The generalised PC can be seen as a static PC computed on weighted data, whereby the variables are weighted according to their common-to-idiosyncratic variance ratio. A variable with a higher common-to-idiosyncratic variance ratio gets a higher weight. The number of aggregates is $r = q\ (s+1)$. Note that the r static factors consist of current and s lagged values of the q dynamic factors. After the common component is estimated, it can be used to compute the conditional h-step ahead forecasts.

2.3 Selecting the Number of Factors

Prior to the estimation of the factor models, the number of static factors, r, and the dynamic factors, q, must be determined. There are a few approaches in determining the number of factors. Bai and Ng (2002) develop some information criteria to determine r, which evaluates the trade-off between goodness-of-fit and over-fitting of a static factor model. After selecting the optimal r, the estimated static factors can be used to determine q, using the information criteria proposed by Bai and Ng (2007). The authors exploit the relationship between the dynamic and static factors to determine q, which are interpreted as the number of primitive shocks driving macroeconomic fluctuations. However, Hallin and Liska (2007) argue that the methodology proposed by Bai and Ng (2007) is based on a restricted dynamic framework and is likely to be overestimated. They develop an information criterion to determine q in a more general dynamic factor model. Another method to determine q is the decision rule of Forni *et al.* (2000) which adds one factor at a time until the additional variance explained by the last dynamic PC is less than a specific value, say 0.05.

3. Data

3.1 Description

The dataset compiled for Brunei consists of 107 quarterly series from 2003Q1 to 2014Q2 (i.e. $T = 46$). The data series consists of both national and transnational indicators, and can be grouped into the following: National accounts (35 domestic variables); Real activity (1 domestic variable, 17 external variables); Government finance (4 domestic variables); Prices (6 domestic variable, 18 external variables); and Financial (17 domestic variables, 8 external variables). External variables account for 41% of the dataset. As noted in Section 1, it is important to include external economic indicators, especially variables pertaining to the global oil market since Brunei's economy is highly oil-dependent. Other external variables that may help to predict Brunei's GDP are the macroeconomic variables of Brunei's main trading partners. The full data series and their sources are listed in Appendix A.

3.2 Data Treatment

Before the dataset is used for estimation and forecasting, the data needs to be transformed. First, for variables that present a seasonal pattern, a seasonal adjustment is applied using the X12-ARIMA procedure. Since the estimation of the dynamic factor models requires stationary time series, all the data series are taken in logarithms and then first-differenced to obtain stationarity, representing growth rates. Exceptions are variables in percentages (e.g. interest rates) or already expressed as changes (e.g. change in stocks), in which the levels are used. The data series are also checked for the presence of outliers. Following Stock and Watson (2005), the outlier adjustment procedure is applied to observations of the transformed series with absolute deviations exceeding six times the interquartile range by replacing them with the median of the preceding five observations. Finally, the data series are normalized to have zero sample mean and unit variance by subtracting their mean and dividing by their standard deviation. This is necessary to avoid overweighting any one series with large variance.

Another issue to deal with is the selection of variables to be included in the dataset. In theory, efficient estimates of the common and idiosyncratic components are obtained asymptotically as the number of variables tends to infinity. However, if a large proportion of the variables provide no additional information and have a low correlation with the variable to be forecast (in this case, GDP growth) then this can impose a cost on predictive accuracy. Boivin and Ng (2006) show that a carefully selected subset of variables can outperform the forecasts from a full dataset. In this regard, variables are pre-selected based on their correlation with GDP growth using a threshold criterion. Only variables with absolute correlation with GDP growth of at least 0.15 are included. This subset contains 34 data series only. Forecasting is performed for both the full dataset and the smaller subset.

4. Forecasting

4.1 Forecasting Strategy

The standard benchmark model is the naïve autoregressive (AR) model of order *p* :

$$y_t = \sum_{j=1}^{p} \phi_j y_{t-j} + \epsilon_t \tag{10}$$

where y_t is GDP growth, the forecast variable of interest, ϕ_j are the AR coefficients, and ϵ_t is the error term. The conditional *h*-step ahead forecast is $\hat{y}_{t+h|t} = \sum_{j=1}^{p} \hat{\phi}_j y_{t+h-j}$, where *p* is the lag selected based on the Akaike Information Criterion (AIC) and $\hat{\phi}_j$ is estimated using ordinary least squares (OLS).

The pseudo out-of-sample forecast evaluation period runs from 2009Q1 through to 2014Q2. This corresponds to about half of the period sample. The quarter-on-quarter GDP growth rate of Brunei is forecast using the AR and the SW and FHLR factor models up to four quarters ahead (i.e. $h = 1, ..., 4$). The out-of-sample forecasts are based on the direct multistep forecasting methodology. Each forecasting model is first estimated using data from 2003Q1 to 2008Q4 and the *h*-step ahead forecasts are then computed. Then the sample is extended by one quarter and the dynamic factors and forecasting models are re-estimated. This recursive procedure is continued until the final set of forecasts is made at the end of the sample period.

The direct forecasting approach differs from the iterated forecast methodology in which future predictions are generated by repeatedly iterating the one-step ahead forecasting equation and replacing unknown values with their predicted values. Using the direct projection method, there is no need to model the evolution of the unobserved factors. Therefore, any misspecification of the *h*-step ahead model will not be transmitted to longer horizon forecasts. Boivin and Ng (2005) show that the direct forecasting methodology works well in factor models.

4.2 Out-of-Sample Evaluation

The forecasting performance is measured using the root mean squared forecast error (MSFE) and the mean absolute forecast error (MAFE):

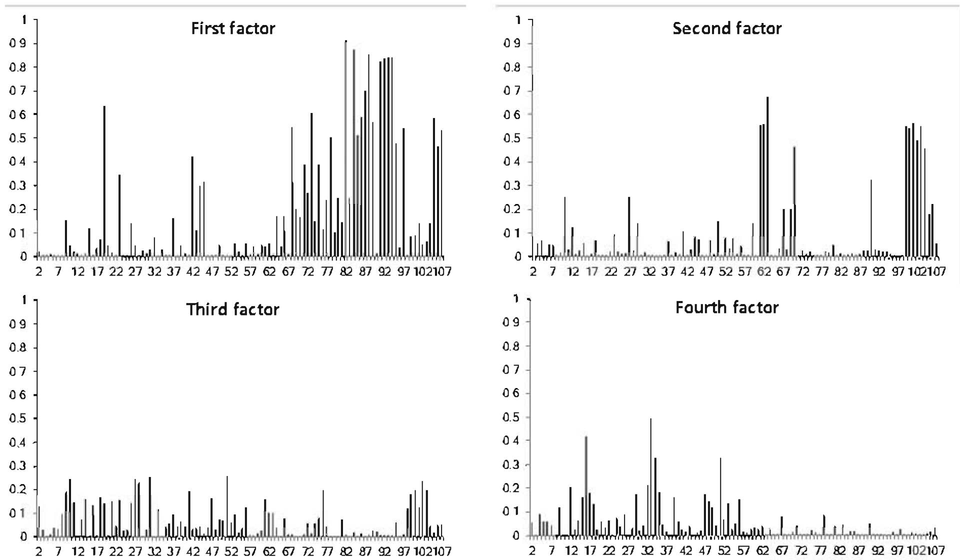


Figure 1. R^2 between the individual data series and each of the four static factors

$$MSFE = \frac{1}{T-h-T_0+1}\sum_{t=T_0}^{T-h}\left(y_{t+h} - \hat{y}_{t+h|t}\right)^2 \tag{11}$$

$$MAFE = \frac{1}{T-h-T_0+1}\sum_{t=T_0}^{T-h}\left|y_{t+h} - \hat{y}_{t+h|t}\right| \tag{12}$$

where y_{t+h} is the actual observed GDP growth at time $t+h$, and $\hat{y}_{t+h|t}$ is the h-step ahead forecast given information up to time t.

The Diebold and Mariano (1995) test is used to test the null hypothesis of equal forecasting accuracy of two competing forecasts based on a loss criterion (e.g. MSFE or MAFE). If the null hypothesis is rejected, a positive test statistic suggests that the first forecast is better, while if it is negative the second forecast is better.

4.3 Common Factors

The number of static factors r is found to be four based on the IC_{p1} and IC_{p2} criteria by Bai and Ng (2002). The first four factors account for 19 per cent, 9 per cent, 7 per cent and 6 per cent respectively (41% cumulatively). Following Stock and Watson (2002a), Figure 1 plots the R^2 of the regressions of the individual series in the full dataset on each of the four factors over the entire sample period. The numbers on the horizontal axis refers to the position listing of the variables in Appendix A.

As seen from the high R^2, the first factor is mostly related to external variables, such as real activity in the advanced economies, commodity prices and oil futures prices, and financial markets. The high R^2 for the domestic variables are GDP and export deflators

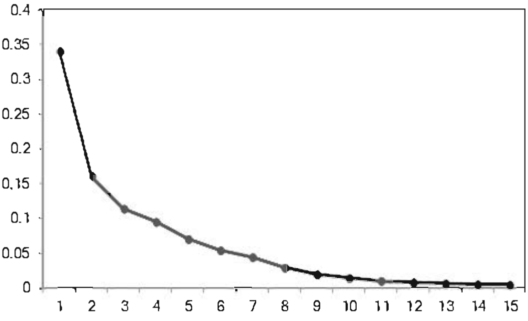
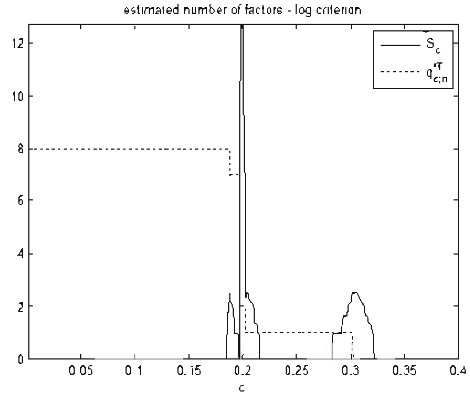


Figure 2. Share of total variance explained by the first 15 dynamic principal components



Notes: Simultaneous plots of $c \mapsto S_c$ and $c \mapsto q^T_{c;n}$ using penalty function p_1 with q_{max}=8,M_T= $0.75\sqrt{T}$ and $IC^T_{2;n}$ criterion.

Figure 3. Identification of the number of dynamic factors as in Hallin and Liska (2007)

as well as bilateral exchange rates. The first factor can thus be interpreted as global demand and supply forces. The second factor reflects monetary policy spillovers, as the high R^2 correspond to the interest rates of various financial instruments in the United States and Singapore, as well as Brunei's domestic interest rates. Note that Brunei imports Singapore's monetary policy as a consequence of its currency board arrangement, and this features in the second factor. The third and fourth factors capture mainly domestic developments but the R^2 of the variables are relatively low, suggesting Brunei's economy is heavily influenced by external forces instead of domestic factors.

The number of dynamic factors varies – using the heuristic approach in Forni *et al.* (2000) the number is three but the Hallin and Liska (2007) information criterion suggests one dynamic factor. Figure 2 plots the share of the total variance explained by the first 15 dynamic PCs. The first four dynamic PCs explain 34 per cent, 16 per cent, 11 per cent and 10 per cent of the total variance respectively (71% cumulatively). Using a threshold marginal contribution to total explained variance of 0.05 suggests choosing q=3. Figure 3 presents

Table 1. Forecast performance of Brunei's quarter-on-quarter GDP growth

	Full dataset h-step ahead forecast				Partial dataset h-step ahead forecast			
	h=1	h=2	h=3	h=4	h=1	h=2	h=3	h=4
AR								
MSFE	0.585	0.559	0.763	0.886	0.585	0.559	0.763	0.886
MAFE	0.192	0.186	0.220	0.244	0.192	0.186	0.220	0.244
Rel. MSFE	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Rel. MAFE	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
SW								
MSFE	0.780	0.621	0.754	0.782	0.529	0.646	0.778	1.294
MAFE	0.224	0.196	0.200	0.243	0.184	0.201	0.203	0.282
Rel. MSFE	1.33	1.11	0.99	0.88*	0.90*	1.16	1.02	1.46
Rel. MAFE	1.16	1.05	0.91*	1.00	0.96	1.08	0.93	1.16
FHLR								
MSFE	0.922	0.687	0.783	0.794	0.517	0.565	0.849	1.267
MAFE	0.230	0.189	0.193	0.233	0.190	0.187	0.208	0.275
Rel. MSFE	1.57	1.22	1.02	0.89*	0.88*	1.01	1.11	1.43
Rel. MAFE	1.20	1.01	0.88*	0.96	0.98	1.00	0.95	1.13

Notes: Full dataset contains 107 variables; partial dataset has 34 variables. The benchmark model is AR(3). The SW and FHLR dynamic factor models have four static and dynamic factors. MSFE values are expressed as x10^{-3} and MAFE as x10^{-1}. * denotes the particular model outperforms the benchmark model based on the Diebold-Mariano test of the same loss criterion at the 10% significance level or better.

the simultaneous plots of $c \longmapsto S_c$ and $c \longmapsto q^T_{c,n}$ to determine q as suggested by Hallin and Liska (2007). Starting from $q_{max} = 8$, the left stable region [0,0.18] represents under-penalisation. The correct value of $q=1$ is revealed in the stable region [0.22,0.28]. However, Forni et al. (2000) note that setting q larger than its true value cannot have dramatic effects on estimation. In the out-of-sample forecasting exercise, r and q are allowed to range from 1 to 4, and the forecasting performance shows that $r = 4$, $r = q$ perform best.

4.4 Forecasting Performance

Table 1 reports the MSFE, MAFE and the relative MSFE and MAFE of the benchmark univariate AR and the SW and FHLR factor models for h = 1, ... 4. The number of autoregressive lags selected is three for the AR model based on AIC. The factor models also contain three autoregressive lags, with $r = 4$, $q = 4$ in both the full dataset and smaller subset. An asterisk (*) denotes the particular model statistically outperforms the benchmark AR model at that forecasting horizon.

Focusing on the full dataset first, the results show that the forecasting errors (MSFE and MAFE) increase for the AR model as the prediction horizon increases. But this is not the case for both the SW and FHLR factor models. The forecasting errors remain roughly unchanged even at four quarters ahead. However, both factor models do not perform as well as the benchmark AR model in forecasting GDP growth up to two quarters ahead. This is due to the presence of many variables with no predictive power at short horizons

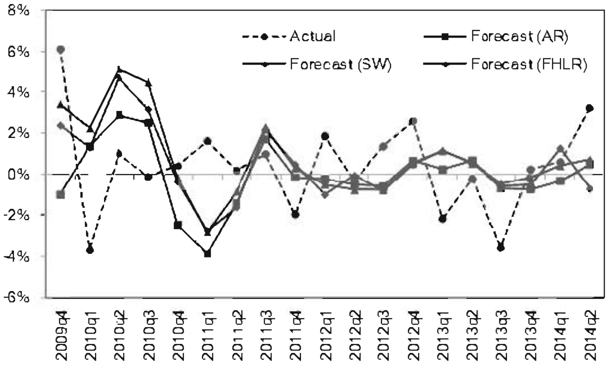


Figure 4. Actual GDP growth and four-quarter ahead forecasts in the full dataset

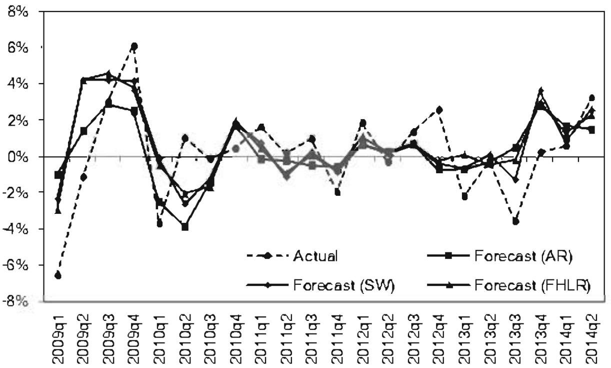


Figure 5. Actual GDP growth and one-quarter ahead forecasts in the partial dataset

which Boivin and Ng (2006) call "oversampling". As the forecast horizon increases, the AR model is less useful as seen in the increasing forecast errors. The factor models, on the other hand, become advantageous due to the information relayed by the various indicators. The relative MSFE and MAFE of the factor models (relative to the AR model) become lower; and at three and four quarters, the factor models statistically outperform the benchmark. Figure 4 plots the four-quarter ahead forecasts of the three models in comparison to actual GDP growth. While the models are unable to get all the direction changes right, looking ahead four quarters, the factor models do a better job with correct direction predictions in 8 out of 19 quarters compared to 7 in the AR model.

If variables are pre-selected, getting rid of those with low correlation with GDP growth, the results for the partial dataset show that both factor models outperform the AR model in the one-quarter ahead forecast. This is displayed in Figure 5. In addition to lower MSFE and MAFE, the factor models correctly predict 16 direction changes out of 22 quarters compared to 14 in the AR model. However, both the forecast errors of the AR and factor models increase with the prediction horizon. At further horizons, the factor models in fact do not forecast as well as the AR model. This is in sharp contrast to the results based on the full dataset. Recall that the variables in the smaller subset are selected based on the contemporaneous absolute correlation with GDP growth of at least 0.15. However, other indicators may contain predictive (leading) information and hence having more variables in the full dataset enables better forecasting performance at further horizons.

5. Conclusion

This paper examines whether factor models can produce accurate forecasts of GDP growth of Brunei Darussalam using a compiled quarterly dataset of 107 indicators. For one- and two-quarter ahead forecasts, the factor models do not perform as well as a standard benchmark univariate autoregressive model due to the presence of indicators with low predictive power at shorter horizons. However, factor models statistically outperform the benchmark at longer horizons as the forecast errors of the benchmark model increase with the prediction horizon while those of the factor models remain relatively unchanged.

Using a subset of the variables selected based on their correlation with GDP growth, the results show that factor models outperform the benchmark for the one-quarter ahead forecast, but the gains are eroded as the horizon increases. This is in contrast to the results from the full dataset. This implies that the wealth of information conveyed by the various indicators in the full dataset provide predictive power for longer horizons. The findings in this paper therefore illustrate the potential usefulness of factor models in forecasting Brunei's GDP growth. Brunei's central bank and government agencies should consider adopting factor models as a complementary forecasting tool to their suite of macroeconomic models.

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Appendix A. List of quarterly series

No.	Data series	Group	Source
	Brunei Darussalam		
1	Gross Domestic Product, constant 2000 prices	National Accounts	DEPD, IMF CR
2	Personal Consumption Expenditure, constant 2000 prices	National Accounts	DEPD, IMF CR
3	Government Consumption Expenditure, constant 2000 prices	National Accounts	DEPD, IMF CR
4	Gross Fixed Capital Formation, constant 2000 prices	National Accounts	DEPD, IMF CR
5	Exports of Goods and Services, constant 2000 prices	National Accounts	DEPD, IMF CR
6	Imports of Goods and Services, constant 2000 prices	National Accounts	DEPD, IMF CR
7	Change in Stocks, constant 2000 prices	National Accounts	DEPD, IMF CR
8	Agriculture, Forestry & Fishery, constant 2000 prices	National Accounts	DEPD, IMF CR
9	Mining, constant 2000 prices	National Accounts	DEPD, IMF CR
10	Manufacturing, constant 2000 prices	National Accounts	DEPD, IMF CR
11	Construction, constant 2000 prices	National Accounts	DEPD, IMF CR
12	Electricity & Water, constant 2000 prices	National Accounts	DEPD, IMF CR
13	Transport & Communications, constant 2000 prices	National Accounts	DEPD, IMF CR
14	Trade, constant 2000 prices	National Accounts	DEPD, IMF CR
15	Finance, constant 2000 prices	National Accounts	DEPD, IMF CR
16	Real Estate & Ownership of Dwellings, constant 2000 prices	National Accounts	DEPD, IMF CR
17	Private Services, constant 2000 prices	National Accounts	DEPD, IMF CR
18	Government Service, constant 2000 prices	National Accounts	DEPD, IMF CR
19	Gross Domestic Product, deflator	National Accounts	DEPD, IMF CR
20	Personal Consumption Expenditure, deflator	National Accounts	DEPD, IMF CR
21	Government Consumption Expenditure, deflator	National Accounts	DEPD, IMF CR
22	Gross Fixed Capital Formation, deflator	National Accounts	DEPD, IMF CR

23	Exports of Goods and Services, deflator	National Accounts	DEPD, IMF CR
24	Imports of Goods and Services, deflator	National Accounts	DEPD, IMF CR
25	Agriculture, Forestry & Fishery, deflator	National Accounts	DEPD, IMF CR
26	Mining, deflator	National Accounts	DEPD, IMF CR
27	Manufacturing, deflator	National Accounts	DEPD, IMF CR
28	Construction, deflator	National Accounts	DEPD, IMF CR
29	Electricity & Water, deflator	National Accounts	DEPD, IMF CR
30	Transport & Communications, deflator	National Accounts	DEPD, IMF CR
31	Trade, deflator	National Accounts	DEPD, IMF CR
32	Finance, deflator	National Accounts	DEPD, IMF CR
33	Real Estate & Ownership of Dwellings, deflator	National Accounts	DEPD, IMF CR
34	Private Services, deflator	National Accounts	DEPD, IMF CR
35	Government Service, deflator	National Accounts	DEPD, IMF CR
36	Oil Production (thousand barrels per day)	Real Activity	EIA
37	Government Oil & Gas Revenue (BND mil)	Government Finance	IMF CR
38	Government Non-Oil & Gas Revenue (BND mil)	Government Finance	IMF CR
39	Government Current Expenditure (BND mil)	Government Finance	IMF CR
40	Government Capital Expenditure (BND mil)	Government Finance	IMF CR
41	Consumer Price Index (2010=100)	Prices	IFS
42	BND per USD, average	Prices	IFS
43	BND per EUR, average	Prices	IFS
44	BND per GBP, average	Prices	IFS
45	BND per JPY, average	Prices	IFS
46	Real Effective Exchange Rate	Prices	Own calculations
47	Total Gross Assets (BND mil)	Financial	IFS
48	Total Reserves Excluding Gold (BND mil)	Financial	IFS
49	Central Bank Net Foreign Assets (BND mil)	Financial	AMBD
50	Central Bank Claims on Other Depository Corp. (BND mil)	Financial	AMBD
51	Central Bank Net Claims on Central Government (BND mil)	Financial	AMBD
52	Banks Net Foreign Assets (BND mil)	Financial	AMBD
53	Banks Claims on Central Bank (BND mil)	Financial	AMBD

54	Banks Net Claims on Central Government (BND mil)	Financial	AMBD
55	Banks Claim on Other Sectors (BND mil)	Financial	AMBD
56	Monetary Base (BND mil)	Financial	AMBD
57	M2 (BND mil)	Financial	AMBD
58	M1 (BND mil)	Financial	AMBD
59	M0 (BND mil)	Financial	AMBD
60	Quasi Money (BND mil)	Financial	AMBD
61	Deposit Rate 3 Months	Financial	AMBD
62	Deposit Rate 12 Months	Financial	AMBD
63	Real Trade-Weighted Interest Rate	Financial	Own calculations
	External		
64	Global Oil Production (thousand barrels per day)	Real Activity	EIA
65	OECD Oil Consumption (thousand barrels per day)	Real Activity	EIA
66	OECD Oil Stocks (million barrels)	Real Activity	EIA
67	OECD Net Oil Imports (thousand barrels per day)	Real Activity	EIA
68	Advanced Economies Industrial Production Index (2010=100)	Real Activity	IFS
69	Advanced Economies Real GDP Growth	Real Activity	IFS
70	Emerging & Developing Economies Real GDP Growth	Real Activity	IFS
71	United States Gross Domestic Product (2010=100, SA)	Real Activity	IFS
72	Japan Gross Domestic Product (2010=100, SA)	Real Activity	IFS
73	Korea Gross Domestic Product (2010=100)	Real Activity	IFS
74	China Gross Domestic Product (2010=100)	Real Activity	IFS
75	United Kingdom Gross Domestic Product (2010=100, SA)	Real Activity	IFS
76	Australia Gross Domestic Product (2010=100, SA)	Real Activity	IFS
77	Singapore Gross Domestic Product (2010=100)	Real Activity	IFS
78	Malaysia Gross Domestic Product (2010=100)	Real Activity	IFS
79	Indonesia Gross Domestic Product (2010=100)	Real Activity	IFS
80	Thailand Gross Domestic Product (2010=100)	Real Activity	IFS
81	Commodity Price, Agricultural Raw Materials (2010=100)	Prices	IFS
82	Commodity Price, All Fuel & Non-fuel (2010=100)	Prices	IFS
83	Commodity Price, Beverages (2010=100)	Prices	IFS
84	Commodity Price, Energy (2010=100)	Prices	IFS
85	Commodity Price, Food (2010=100)	Prices	IFS
86	Commodity Price, Metals (2010=100)	Prices	IFS
87	Commodity Price, Non-Energy (2010=100)	Prices	IFS
88	Commodity Price, Crude Oil (USD per barrel)	Prices	IFS
89	Commodity price, Coal (USD per metric ton)	Prices	IFS
90	Commodity Price, Natural Gas (USD per million BTU)	Prices	IFS
91	Crude Oil Futures Contract 1, Cushing OK (USD per barrel)	Prices	EIA
92	Crude Oil Futures Contract 2, Cushing OK (USD per barrel)	Prices	EIA
93	Crude Oil Futures Contract 3, Cushing OK (USD per barrel)	Prices	EIA
94	Crude Oil Futures Contract 4, Cushing OK (USD per barrel)	Prices	EIA
95	Advanced Economies Consumer Price Index (2010=100)	Prices	IFS
96	Emerging & Developing Economies Consumer Price Index (2010=100)	Prices	IFS
97	United States Consumer Price Index (2010=100)	Prices	IFS
98	Singapore Consumer Price Index (2010=100)	Prices	IFS

99	United States Monetary Policy Rate	Financial	IFS
100	United States Treasury Bills Rate	Financial	IFS
101	United States Government Bonds Rate	Financial	IFS
102	Singapore Monetary Policy Rate	Financial	IFS
103	Singapore Treasury Bills Rate	Financial	IFS
104	Singapore Government Bonds Rate	Financial	IFS
105	S&P Index (2010=100)	Financial	IFS
106	NASDAQ Index (2010=100)	Financial	IFS
107	MSCI Emerging Markets Stock Price (USD)	Financial	MSCI

Source: AMBD – Authoriti Monetari Brunei Darussalam; DEPD – Department of Economic Planning and Development, Brunei Darussalam; IMF CR – IMF Country Reports; EIA – US Energy Information Administration; IFS – IMF International Financial Statistics; MSCI – Morgan Stanley Capital International.